ACCESS TO QUALIFIED MATHEMATICS TEACHERS FOR ALL STUDENTS

SEPTEMBER 2025

$x \neq 0$

$y = mx + b$

$\sqrt{a}$

CRPE
REINVENTING PUBLIC EDUCATION

ASU Mary Lou Fulton College for Teaching and Learning Innovation
Arizona State University

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About the Center on Reinventing Public Education

The Center on Reinventing Public Education (CRPE) is a nonpartisan research organization at Arizona State University's Mary Lou Fulton College for Teaching and Learning Innovation. Since 1993, we have studied innovative, evidence-based solutions to improve public education. We believe public education is a goal—to prepare every child for citizenship, economic independence, and personal fulfillment—rather than a set of institutions. From this foundation, we strive to understand and advocate for necessary changes in policy and practice to meet the needs of every student.

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> "For all students to become mathematically proficient, major changes must be made in mathematics instruction, instructional materials, assessments, teacher education, and the broader educational system." (National Research Council, 2002, p. 1)
>
> "The delivery system in mathematics education [...] is broken and must be fixed. This is not a conclusion about any single element of the system. It is about how the many parts do not now work together to achieve a result worthy of this country's values and ambitions." (National Mathematics Advisory Panel, 2008, p. 8)

While there is much to celebrate in mathematics education, both quotes above highlight that experts have long voiced concerns about the field, advocating for improvements in curriculum content, teacher preparation, instructional practices, assessment, and policy.[1] As the COVID-19 pandemic worsened existing inequities that contributed to declines in student achievement, various news articles and research reports raised the issue of a possible national teacher shortage, frequently noting increases in local vacancy and attrition rates. Indeed, public concern about a teacher shortage spiked at the beginning of the 2022-23 school year, leading to local and state education leaders' renewed interest in bolstering, augmenting, and maintaining their teaching workforce. The much-anticipated release of the latest National Assessment of Educational Progress (NAEP) results on January 29, 2025, did little to dispel ongoing concerns, as only 39% of fourth graders and 28% of eighth graders performed at or above the NAEP "Proficient" benchmark in mathematics. Anticipating public debate on improving student achievement, the Center on Reinventing Public Education (CRPE) convened a panel of mathematics experts who identified **student access to qualified mathematics teachers** as a priority issue in mathematics education, a subject that also stands to inform the latest iteration of this debate.

At the outset of this report, we'd like to acknowledge that the sheer number of challenges and strategies intended to solve them makes it difficult to provide a straightforward answer to this deceptively simple question: **Do all students have access to qualified mathematics teachers who can deliver effective mathematics instruction?**

1. National Mathematics Research Panel, 2008.

A thorough answer must establish what it means for a mathematics teacher to be "qualified," what "effective" mathematics instruction should entail, and the extent to which available information applies to "all" students, considering important subpopulations such as high- and low-achieving students, students with disabilities, or students from different socioeconomic, linguistic, or cultural backgrounds. While the extensive length of such an answer may be overwhelming for a general audience, this technical report aims to contribute to the discussion on improving students' mathematics achievement by providing a landscape overview of the access issue, highlighting **three necessary (but not sufficient) conditions** that must be met before we can answer this question affirmatively. As such, student access to qualified mathematics teachers who can deliver effective mathematics instruction is predicated on three conditions:

- **Condition #1**: We have enough qualified mathematics teachers to fill open teaching positions.
- **Condition #2:** We have mathematics teachers who demonstrate the knowledge, skills, and practices to deliver effective mathematics instruction.
- **Condition #3:** We have mathematics teachers who use high-quality instructional materials to implement a focused, coherent curriculum aligned to state standards.

In this report, we present available information on each condition, highlighting current challenges and strategies in place to address them. Consequently, the identified approaches for tackling key challenges under each condition should not be interpreted as recommendations. This landscape overview aims to assess the status quo and lays the groundwork for subsequent, more detailed briefs that critically analyze specific challenges and solutions.

INTRODUCTION

Access to qualified teachers matters. It is a key component to student success in every subject, including mathematics. We know a substantial amount of students' math achievement gains in a given school year can be directly attributed to differences in teachers,[2] which can lead to measurable differences between students that can compound dramatically over time. When students have consistent access to qualified mathematics teachers, their achievement gains enable them to build a strong foundation, demonstrating fluency with whole numbers, fractions, and the basic aspects of geometry and measurement. Subsequently, students are prepared to successfully take on advanced coursework, such as Algebra, and expand their knowledge and skills in related fields. Students with access to qualified mathematics teachers from a young age show improved logical reasoning, critical thinking, and problem-solving, making them more likely to pursue higher education and excel in competitive, higher-paying fields.[3] Success in mathematics allows students to meet future workforce demands and pursue in-demand careers in information technology, cybersecurity, and artificial intelligence.

While the importance of qualified mathematics teachers promoting higher student achievement remains clear, our students' performance on national and international assessments has consistently shown that a majority of students (a) are not proficient in mathematics by fourth grade, (b) become less proficient over time, and (c) lag behind the performance of students in other developed countries. Proficiency gaps also pertain to some students more than others, including Black and Latino students, female students, and students with disabilities. The disruptions of the COVID-19 pandemic erased most of the modest performance gains made over the past decade, which has prompted numerous news reports to examine the role of a worrisome trend: more teachers are leaving the profession, while fewer college students are interested in the teaching profession. This raises questions about the extent to which qualified mathematics teachers are available to all students and whether they have the capacity to promote student success in mathematics.

In light of news reports suggesting a national teacher shortage, the limited

2. Based on an experimental design, Nye, Konstantopoulos, and Hedges (2004) found that the difference in achievement gains in mathematics between having a 50th percentile teacher (an average teacher) and a 90th percentile teacher (a very effective teacher) is close to half a standard deviation (0.46).

3. A substantial body of research exists about the importance of students' mathematics achievement, especially from an early age, for later achievement across a broad range of educational outcomes (e.g., Claessens & Engel, 2013; Adelman,2006; Attewell & Domina,2008; Byun, Irvin, & Bell,2015; Kim, DesJardins, & McCall,2015) and economic outcomes (e.g., Gaertner, Kim, DesJardins, & McClarty, 2014; Rose & Betts, 2004).

data available paints an incomplete picture of many state and district leaders facing challenges filling open positions with certified teachers and providing those teaching with high-quality instructional materials and additional training to implement effective mathematics instruction. Unfortunately, school leaders also face the challenge of selecting high-quality curricula amidst an unprecedented number of textbook bans. They must further decide on in-service professional development while navigating ongoing debates among researchers about which practices qualify as effective mathematics instruction. At the same time, the research-to-practice gap continues to loom large in the area of mathematics, with several research studies suggesting that some teachers continue to implement instructional practices that are either disproven or lack sufficient research to support their use.[4] Yet researchers themselves appear to be short of a clear consensus on what constitutes effective mathematics instruction, debating the "science" behind certain teaching practices, including some recommended by national organizations.

In summary, the undeniable significance of mastering mathematics for our students' career and life outcomes and our nation's competitiveness stands in stark contrast to their performance on national and international mathematics assessments. This discrepancy raises concerns about reports indicating shortages of qualified mathematics teachers and ongoing debates among researchers regarding the knowledge, skills, and practices these educators should possess in the classroom. We begin by examining the first access condition and what is known about the supply of qualified mathematics teachers, noting key challenges and current strategies.

4. Hott et al., 2019; Peltier et al., 2021; van Dijk & Lane, 2020.

CONDITION #1: DO WE HAVE ENOUGH QUALIFIED MATHEMATICS TEACHERS TO FILL OPEN TEACHING POSITIONS?

While we do not systematically collect data on the nation's teacher workforce, available information suggests that there are not enough fully certified mathematics teachers to fill all open teaching positions.

To address this question, we must begin by defining the term "qualified" in a way that is measurable and commonly found in public reports supplied by local and state education agencies. Researchers who examine the teacher workforce typically differentiate between (a) **certified teachers** who teach in their endorsed area (e.g., a licensed high school mathematics teacher teaching a ninth grade Algebra class), (b) certified **"out of field" teachers** who teach outside of their endorsed area (e.g., a licensed physical education teacher teaching English), and (c) not fully certified **teachers who teach based on an emergency credential**, waiver, or some other type of provisional license. Aligning with these basic definitions, we considered data on teachers fully certified in their area as information on the availability of qualified teachers. Next, we examined data on the overall teacher workforce, followed by specific data on mathematics teachers.

Since the U.S. Department of Education (ED) does not collect teacher workforce data in a systematic and centralized manner, we currently lack a national database, making it challenging to assess the extent of the teacher shortage with any meaningful specificity. Consequently, researchers must rely on aggregating publicly available information published by various local and state education agencies. The lack of data fuels a debate among researchers. Some argue that we face a national teacher shortage, while others believe we are only encountering localized shortages in certain districts, states, and subject areas.

Access to data regarding the teacher workforce, which allows for insights into where and what type of teacher talent is available and needed, is not a new challenge. However, this issue has worsened since the pandemic, as teachers have faced unprecedented levels of stress and burnout. This has raised additional questions about increased teacher turnover rates and lower enrollment in teacher preparation programs. As such, we identify three key challenges that create significant barriers to ensuring that we have enough qualified mathematics teachers:

1. **Teacher workforce data**
2. **Teacher turnover**
3. **Teacher pipeline**

Challenge #1: We lack systematic and centralized data collection on the teacher workforce.

This hinders our ability to fully understand important details about the teacher workforce, both within and across states, making it challenging to identify where specific teachers are needed and for which subjects and grades.

We have limited knowledge about the teacher workforce and students' access to qualified mathematics teachers at a national level. For instance, a study by the National Council on Teacher Quality found that only 16 states (32%) published major components of teacher workforce data on their **state websites**. They observed that only 12% of states disaggregate data on teacher vacancies by subject area, and just 53% provide teacher attrition data by subject area.

To understand the extent to which states make teacher workforce data available in their **school report cards**, we conducted a state-by-state analysis of publicly available school report cards. Specifically, we assessed whether states made data on (a) teacher demographics, (b) vexperience, (c) certification, (d) attrition, and (e) vacancies readily available to parents and other stakeholders. For more information, please visit the CRPE website for a summary report and interactive dashboard. Regarding vacancies, only two states (Nevada and South Carolina) posted information on teacher vacancies on their most recently available school report cards. Through a more extensive search of each state education agency's public website, we were able to locate state-level data on teacher vacancies in an additional seven states. In total, only nine states (18%) publicly reported the number of school vacancies on their school report cards. The earlier report by the National Council on Teacher Quality (NCTQ, 2021) identified 16 states (32%) that published major components of teacher workforce data on their state website.

In the absence of a national database, what we know about student access to qualified teachers must be pieced together from state-level reports. Fortunately, several research groups have worked on this issue and published reports with estimates related to critical teacher shortage areas, including the Learning Policy Institute and Nguyen, Lam, and Bruno(2022).[5] For the 2024-25 school

5. Nguyen and colleagues also operate a website that provides updated data.
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year[6], available estimates indicate that students are being taught by between **365,044 and 400,000 teachers not fully certified** for their assignments, including teachers with emergency credentials and teachers teaching "out of field." These numbers represent about 11% to 12% of the total number of public school teachers. Regarding vacancies, the most comprehensive estimate[7] indicates a minimum of **49,000 unfilled teaching positions**, with North Carolina, Florida, Illinois, and Virginia at the top of the vacancy chart (i.e., over 3,000 vacancies). To improve interpretations and allow for comparisons, Nguyen and colleagues further estimated regional vacancy rates (i.e., vacancies per 10,000 students), which changed vacancy rankings. Mississippi moved to the top based on vacancy rate, with an estimated 62 vacancies per 10,000 students, followed by West Virginia, Maine, North Dakota, and North Carolina—all above 30 vacancies per 10,000 students.

As helpful as these estimates are, they do not reveal **within-state differences**, such as differences between districts or schools (e.g., elementary, rural, Title I), or provide additional details on **subject-specific vacancies**. As such, we cannot answer important questions, including the extent to which students have access to certified mathematics teachers or if students in Title I schools are taught by a disproportionate number of emergency credentialed or "out of field" mathematics teachers. What we do know is that, historically, mathematics teaching positions have been among the hardest to fill. Based on a review of available data from the School Pulse Panel, a survey of public K–12 schools on high-priority, education-related topics conducted by the National Center for Education Statistics, we found that the percentage of **schools reporting being understaffed in mathematics** has steadily increased over the last three years from 31% in August 2022 to 34% in October 2024. The percentage of schools reporting one or more vacancies in mathematics grew to 36% in March 2024.

6. It is important to note that the various data points necessary for these estimates are not always current for the 2024-25 school year. Instead, the estimates for the current school year are based on the most current state data available.

7. The estimate reported by Nguyen and colleagues was based on information from 45 states + DC. The Learning Policy Institute reported a minimum of 41,902 vacancies based on 30 states + DC.

When disaggregated by lower and higher school neighborhood poverty, schools in higher-poverty neighborhoods consistently reported higher percentages of understaffing than schools in lower-poverty neighborhoods (see Figure 1). Both school types reported an increase in vacancies between June 2022 and March 2024, with **39% of schools in higher-poverty neighborhoods reporting one or more teacher vacancies in mathematics.**

Figure 1: Percentage of Schools Reporting One or More Teacher Vacancies in Mathematics, by Neighborhood Poverty Level

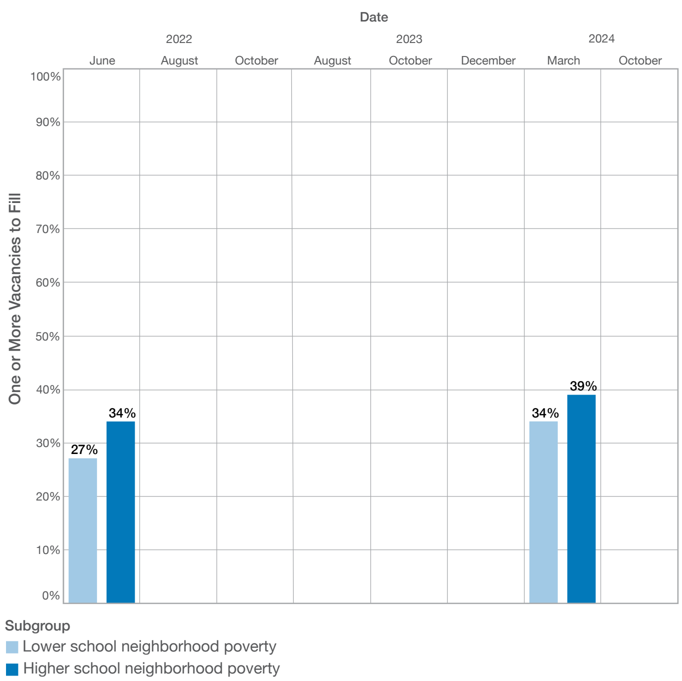
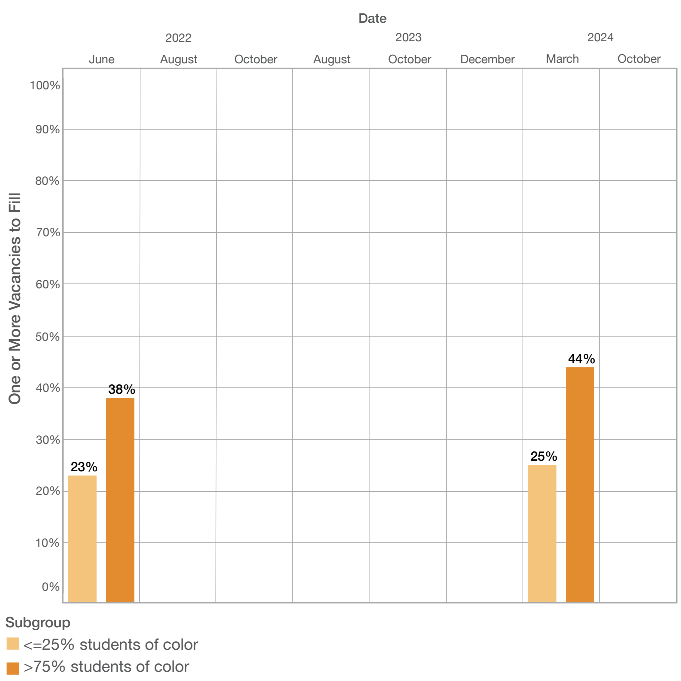


Figure 2: Percentage of Schools Reporting One or More Teacher Vacancies in Mathematics, by Student Racial Demographics



When disaggregated by the percentage of students of color, schools with more than 75% of students of color consistently reported higher percentages of understaffing than schools with 25% or fewer students of color (see Figure 2). Both school types reported increased vacancies between June 2022 and March 2024, with **44% of schools with more than 75% of students of color reporting one or more teacher vacancies in mathematics.**

Additional disaggregations by school level, region, locale, and school size based on the latest survey from October 2024 indicated school leaders who reported the highest percentages of feeling understaffed in the area of math came from **high schools**, those located in the **South**, schools situated in **cities and towns**, and **large schools** with 1,000 or more students. For additional information and access to an interactive data dashboard using the School Pulse Panel data set, please visit the Math Hub on the CRPE website.

A nationally representative school leader survey from the National Center for Education Statistics from 2023[8] confirmed the trend in the School Pulse Panel data, with a consistent increase in the percentage of school leaders reporting difficulties filling open teaching positions across all 12 reporting categories between the 2011-12 and 2020-21 school years. For mathematics, **the percentage of school leaders reporting difficulties filling open teaching positions increased from 19% to 32%**.

Challenge #2: Teacher turnover remains an issue as we continue to recover from the effects of the pandemic.

While the available data does not indicate a mass exodus of teachers following COVID, teacher turnover remains expensive, and its effects are not uniform, disproportionately affecting specific areas such as mathematics and schools in low-income neighborhoods.

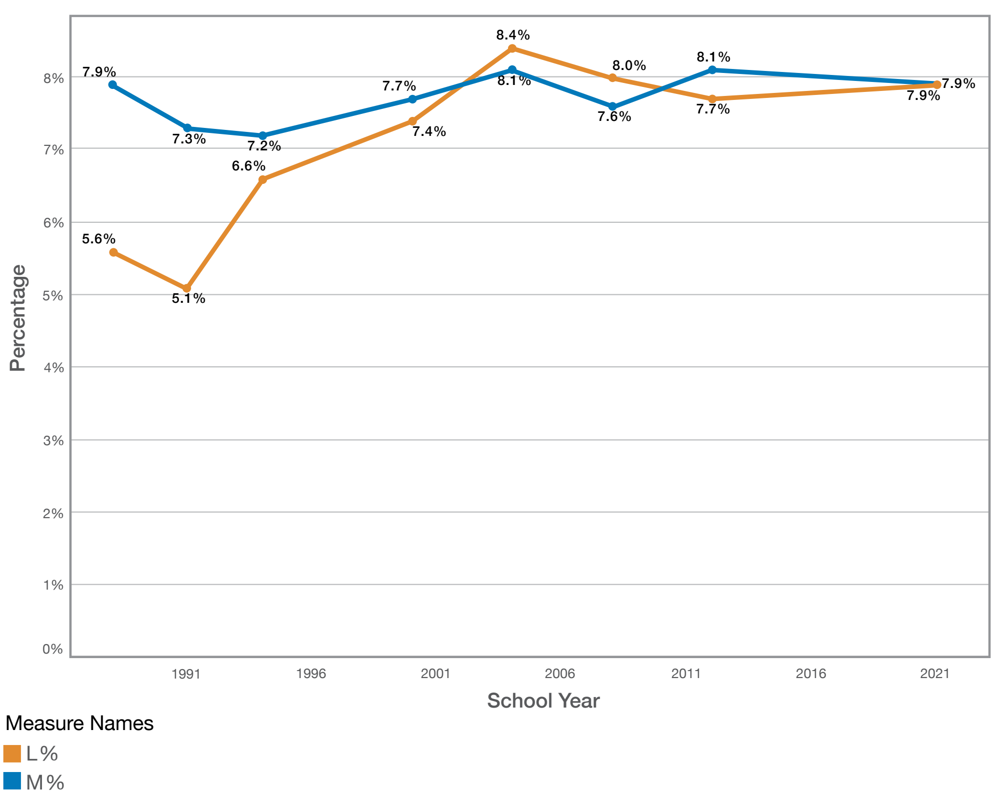
Since the pandemic, numerous news reports have highlighted an unprecedented number of teachers leaving the profession as many schools struggle with increased absenteeism among both students and teachers, challenges in student socioemotional and behavioral development, and a rise in the percentage of students seeking mental health services. Given the negative impact of high teacher turnover on student achievement and an **estimated replacement cost of nearly $25,000 per teacher**,[9] concerns about teacher turnover are well-founded. A 2023 research report by the RAND Corporation based on a national survey of 300 district and charter network leaders indicated teacher turnover had increased four percentage points above pre-pandemic levels, reaching **10% nationally at the end of the 2021-22 school year**. Among the hardest hit were urban districts, high-poverty districts, and districts predominantly serving students of color.

8. See Taie & Lewis, 2023.

9. See the interactive Learning Policy Institute's turnover cost calculator.

The National Center for Education Statistics published its findings on teacher attrition and mobility in December 2023, based on a follow-up to the National Teacher and Principal Survey. Teacher turnover data typically includes three types of teachers: (a) "leavers," who are teachers that leave the profession from one school year to the next; (b) "stayers," who are teachers that stay at the same school; and (c) "movers," who are teachers that move to a different school. Based on eight available data points, starting with the 1988-89 school year and ending with the 2021-22 school year, the vast majority of teachers had stayed at their schools, ranging between 83.5% for the 2004-5 school year and 87.6% for the 1991-92 school year. Looking at movers and leavers, the percentages have come closer together over time, with both ending at **7.9% for the 2021-22 school year** (see Figure 3). The peak for teachers leaving the profession occurred in the 2004-5 school year when 8.4% of teachers reported leaving the profession. Based on teacher characteristics, the highest percentages of "leavers" within the categories of age, sex, race/ethnicity, base salary, education, and total years of experience were 55 years or older (16.1%), female (8.2%), Black or African American (10.7%) and those who had a base salary of less than $40,000 (10.7%), education higher than a master's degree (11.6%), and 15 or more years of teaching experience (8.8%).

Figure 3: Percentage of Public School Teachers Who Left the Profession



Of those teachers whose main teaching assignment was mathematics, **8.1% reported leaving the profession** for the 2021-22 school year, ranking behind other teaching assignments, such as career or technical education (13.9%), bilingual education (11.9%), and special education (8.5%).

While existing data are insufficient to examine differential attrition in different school and classroom settings, available data suggest that teacher attrition contributes to teacher shortages and represents a core issue in efforts to provide all students with access to mathematics teachers. However, the changes in attrition during the pandemic and its aftermath appear to be within the normal range of fluctuation, falling short of indicating a nationwide mass exodus of teachers.[10] As such, the previously identified contributors driving turnover remain relevant, including poor working conditions, dissatisfaction, and school management issues—with high-poverty schools suffering the most due to challenging work environments.[11] Given that the mathematics and science fields are particularly affected because the new teacher supply closely matches attrition, we examine teacher supply next.

Challenge #3: The number of students moving through the teacher preparation pipeline has decreased.

While public school enrollment has not declined significantly compared to over a decade ago, the number of teaching graduates has, resulting in a reduced supply amid unchanged demand.

Title II of the Higher Education Act of 1965 requires all teacher preparation providers and states to report key information about their teacher preparation programs (TPPs) to the U.S. Department of Education (ED), including TPP enrollment and completion data. This data can indicate the supply of future teachers in the pipeline and is seen as reflecting the degree of general interest in the teaching profession.

The latest national Title II data for the 2022-32 academic year indicated that **586,805 students enrolled** in a TPP and **148,931 students completed** their TPP across 2,218 TPP providers offering 25,009 programs.

10. Goldhaber & Theobald, 2023.

11. Ingersoll, 2025.

Looking at longitudinal enrollment data (see Figure 4),[12] it becomes clear that enrollment in TPPs rapidly declined as we entered the 2010s, dropping from 621,898 students for the 2011-12 academic year to a low of 417,585 for the 2014-15 academic year, an over 32% decrease. Since then, TPP enrollment has increased, briefly stabilized around 600,000, and then decreased to 586,805 for the 2022-23 academic year. Compared to over a decade ago, the 2022-23 TPP enrollment total represents 94% of the 2011-12 enrollment total.

Figure 4: Total Enrollment in Teacher Preparation Programs (TPPs)

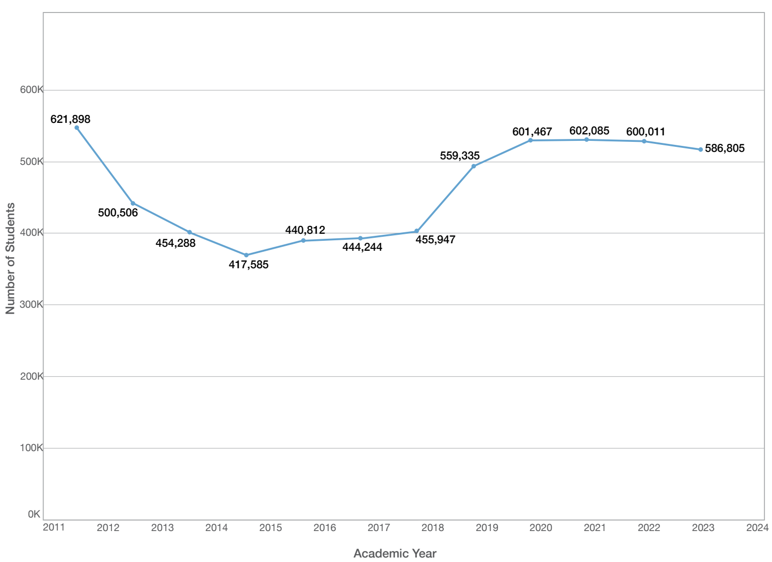
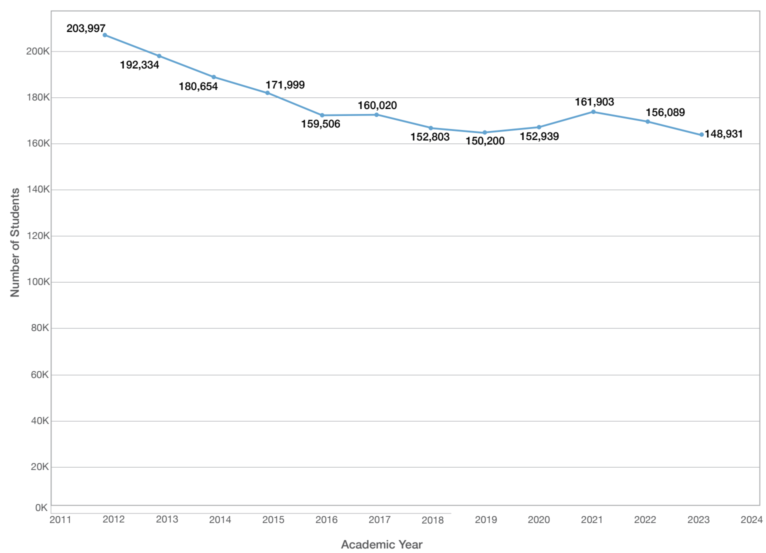


Figure 5: Total Completion of Teacher Preparation Programs (TPPs)



Looking at longitudinal TPP completion data (see Figure 5), a similar decline becomes noticeable, dropping from 203,997 completers in the 2011-12 academic year to a historic low of 148,931 in the 2022-23 academic year, a 27% decrease Since then, TPP completion has increased only modestly. Compared to over a decade ago, the 2022-23 TPP completion total represents 73% of the 2011-12 completion total.

When examining enrollment and completion numbers by state, the observed nationwide decline is also evident in most states. However, some states contradicted the national trend. Between 2012-13 and 2019-20, enrollment

12. Between 2017-18 and 2018-19 the definition of "enrolled students" became more inclusive, counting those students who were graduating during the reported academic year as enrolled.

numbers increased in Washington, the District of Columbia, Nevada, and Texas. During the same timeframe, Nevada and the District of Columbia were also able to increase their completion numbers.

Enrollment and completion numbers can be disaggregated by the type of TPP, which can be categorized into traditional programs, alternative programs based at institutions of higher education ("alternative IHE"), and alternative programs not affiliated with higher education institutions ("alternative non-IHE"). Traditional TPPs are typically offered by higher education institutions and often culminate in a bachelor's or master's degree. However, it's important to note that traditional programs can also operate outside of higher education institutions, while alternative programs can be found within them. Moreover, the content and structure of TPPs vary greatly, even within these classifications. Students enrolled in alternative programs generally teach while completing their training.[13] In comparison, traditional programs usually necessitate more coursework focused on teaching methods and are more likely to include student-teaching placements.

The vast majority of students interested in teaching enroll in traditional programs (around 70%), followed by alternative non-IHE programs (around 20%) and alternative IHE programs (around 10%). However, this data can vary significantly from state to state. For 2019-20, for example, Texas reported that a stunning 75% of TPP students were enrolled in alternative non-IHE programs.

When disaggregating by TPP type, it becomes clear that **traditional programs account for the decline in national TPP enrollment**. Comparing 2019-20 to 2012-13 enrollment data, traditional program numbers were 31% lower. In contrast, enrollment increased by 35% in alternative IHE programs and 117 % in alternative non-IHE programs during the same time.

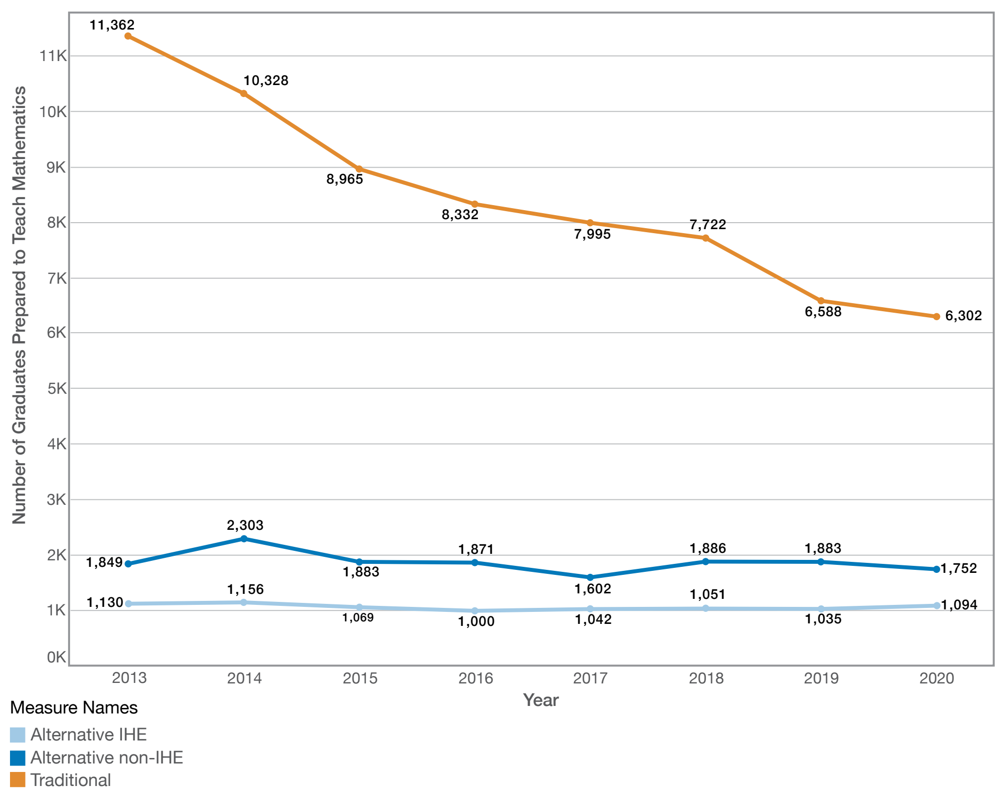
Since the TPP completion numbers provide useful information for understanding the supply of teachers, we looked into available completion data disaggregated by TPP type. Based on 2019-20 data, the total number of 152,939 students who completed TPPs can be disaggregated into 117,567 who completed traditional programs (77%), 17,552 who completed alternative IHE programs (11%), and 17,820 who completed alternative non-IHE programs (12%). When we examine completion numbers by subject area, mathematics ranks fifth out of 13 subject areas across all three program types, accounting for **6% of all graduates**. Disaggregated by

13. See U.S. Department of Education, 2020.

TPP type, the respective percentages for individuals who graduated prepared to teach mathematics were 5% of traditional graduates, 7% of alternative IHE graduates, and 7% of alternative non-IHE graduates. The top three subject areas are elementary education, special education, and early childhood education, regardless of program type. Combined, these three subject areas represented 72% of traditional graduates, 65% of alternative IHE graduates, and 59% of alternative non-IHE graduates.

When examining the trends from 2012 to 2020, the number of graduates prepared to teach mathematics has remained stable for alternative IHE and non-IHE programs. For traditional programs, the **number of graduates prepared to teach mathematics has consistently declined** from 11,362 in 2012-13 to 6,302 in 2019-20 (see Figure 6), representing a decrease of 45%. Between 2012-13 and 2019-20, **mathematics experienced the largest percentage decrease among all subject areas for traditional graduates.** During the same period, the percentage decrease of mathematics graduates across all three program types was 36%. As a percentage of all program completers, the percentage of graduates prepared to teach mathematics declined from 7.46% in 2012-13 to 5.98% in 2019-20.

Figure 6: Number of Teacher Preparation Program Completers Prepared to Teach Mathematics, by Program Type



TAKEAWAYS

1. Teacher workforce data are not collected consistently and comprehensively within and across states, making it difficult to assess and address staffing challenges with the precision needed to develop targeted solutions, including by grade, subject, or school.

2. Teacher vacancies vary by state, with estimated vacancy rates indicating that Mississippi, West Virginia, Maine, North Dakota, and North Carolina have some of the highest vacancy rates in the nation (i.e., above 30 vacancies per 10,000 students).

3. The percentage of schools reporting math staffing challenges has increased over the past few years. The latest data indicate that the percentage of schools reporting one or more math vacancies has increased to 36%. This percentage jumps even higher for schools in higher-poverty neighborhoods (39%) and schools educating mostly students of color (44%).

4. Annual teacher turnover is estimated to range between 8% and 10%, which remains below other fields such as technology (13-20%), manufacturing (15-25%), and healthcare (18-30%). Based on data collected by government surveys, the percentage of teachers leaving the profession has remained fairly consistent, around 8%, with a peak of 8.4% in 2004-5. For teachers whose main teaching assignment was mathematics, 8.1% reported leaving the profession for the 2021-22 school year.

5. Teacher pipeline data indicate a notable decline in enrollment and completion that began over a decade ago, largely attributable to traditional TPPs, which enroll about 70% of TPP students. In contrast, alternative IHE TPPs have consistently increased their enrollment numbers. Based on the number of students completing their programs, the 2021-22 total shows a 23% decrease compared to the total a decade ago. The number of graduates prepared to teach mathematics also declined overall, largely attributable to a decline in traditional TPPs, and this decline was more pronounced for mathematics than other subject areas.

Common strategies to address the teacher shortage involve increasing the number of new teachers in the pipeline and enhancing the retention of experienced educators.

Even though public school enrollment dropped below 50 million students for the first time in nearly a decade during the 2020-21 school year, it has remained fairly stable, with the 2023-24 enrollment down by less than 400,000 students compared to the 2012-13 enrollment (0.7% decrease). While public school enrollment has not declined significantly, the number of teaching graduates certainly has, creating a situation of decreased supply amidst relatively unchanged demand. In response, the main approaches for addressing this supply-and-demand predicament can be categorized into strategies aimed at (a) increasing the supply of employable teachers and (b) keeping those already employed. Below, we outline some of the most common strategies currently in use for addressing teacher shortages.

Strategy Example: Alternative teacher preparation programs offer new pathways into the teaching profession.

Alternative preparation programs can lower entry barriers to the teaching profession and attract prospective candidates outside the traditional teacher preparation pathways.

As the decline in interest in the teaching profession has moved in tandem with a decrease in occupational prestige and job satisfaction,[14] policymakers, thought leaders, boards of education, as well as colleges and universities have focused increasingly on finding ways to reduce entry barriers to the profession, offering more flexible ways to become a teacher through so-called alternative certification programs. For teachers prepared via traditional pathways, their training to become qualified teachers typically begins with education coursework as part of their major, culminates in student teaching, and ends in employment after they complete all coursework and certification requirements. For teachers prepared via alternative pathways, their training typically builds upon an existing (noneducation) degree and allows them to teach before completing their certification requirements. These alternative programs can be run by postsecondary institutions (i.e., alternative IHE programs) or organizations and actors outside postsecondary institutions (i.e., alternative non-IHE programs).

Alternative-route TPPs are a topic of much debate concerning quality, cost, preparation, and professionalism. These issues are significant when evaluating the overall benefit of various pathways into teaching. Despite these concerns, these alternative programs have opened doors for many prospective

14. See Kraft and Lyon's 2024 paper on the rise and fall of the teaching profession.

teachers and continue to be a popular way to increase the number of teachers available to students. Alternative-route TPPs can be viewed as a pragmatic response to persistent teacher shortages, particularly in urban classrooms.[15] These programs offer nontraditional and often accelerated pathways to state teaching licensure, with variations in infrastructure, design, type of participants, and outcomes.[16] Alternative IHE programs are found in nearly every state, with Texas leading enrollment and completion numbers through these pathways.[17] Available information on alternative non-IHE programs is presently very limited. A recent report by the Center for American Progress indicated that non-IHE programs exist in 32 states and Washington, D.C., with for-profit organizations enrolling about 68% of all students in non-IHE programs. Given previous reports about the deceptive and harmful practices of for-profit higher education programs, more research is needed on the extent to which these private entities serve the needs of their students and communities.

Alternative routes to teaching should not be understood as a single, uniform intervention; there is as much variation within alternative programs as there is between traditional and alternative programs. However, it is clear that programs with lower entry barriers and shorter training periods have made teaching a more accessible pathway for many individuals. Most alternative IHE programs are based on a partnership between local education agencies and institutions of higher education, which collaborate to provide education coursework and supervised field experiences.[18] While the previously discussed enrollment and completion numbers in these programs indicate their consistently increasing popularity, research studies on the effectiveness of these programs in relation to outcomes such as teacher knowledge, retention, efficacy, and growth have reported mixed results. Available evidence suggests that effective alternative certification programs tend to focus on placing candidates in supportive school environments with strong leadership, collegiality, and adequate resources. They prioritize selecting well-educated candidates, enhancing subject-matter knowledge, and valuing prior classroom experience. These programs provide tailored and timely coursework, assign trained mentors to support candidates, and offer frequent feedback through classroom observations, thus representing the characteristics of effective TPPs in general.[19]

15. See Ng, 2003.

16. Rosenberg et al., 2023.

17. IES, 2024; King & Yin, 2022.

18. Day, 2022.

19. For a comprehensive overview of alternative certification programs, see Constantine, et al., (2009).

Strategy Example: Financial and other incentives can attract teachers into high-need areas and enhance retention.

Financial incentives can be an effective strategy for attracting more teachers to high-need subjects like mathematics and special education, at least temporarily. Some states have utilized visa exchange programs for international teachers to address acute staffing shortages.

Teacher Incentives. Using teacher incentives to recruit and retain teachers is a common strategy employed across the U.S. and abroad, particularly in high-need schools and subjects. The list of incentives is long, including financial incentives (e.g., signing and retention bonuses, pay supplements, loan forgiveness, tuition subsidies, housing assistance) and nonfinancial incentives (e.g., induction and mentoring programs, expanded career and leadership advancement options, flexible working conditions, reduced teaching workloads). Research on the effectiveness of these incentives for recruiting and retaining teachers is limited, partly due to the complexity of the variables affecting these teacher decisions. Recent studies have identified several variables and their relation to recruitment and retention, including teacher preparation, the school's organizational practices, peer communication among staff, administrative support, individual characteristics, community, and working conditions.[20] A 2020 review of 120 recruitment and retention interventions highlighted a lack of robust studies but identified the strongest evidence for financial incentives. Specifically, the authors noted the most promising evidence related to financial incentives was for (a) recruiting new teachers into the teaching profession and (b) recruiting more teachers into high-performing or high-poverty schools. Unfortunately, the available evidence does not support financial incentives as an effective means of retaining teachers over the long term. Financial incentives, which are often tied to contractual agreements, appear to be a short-term solution only, with the majority of studies reporting that teachers leave once the incentives expire. Sustained financial incentives, such as targeted salary policies, can have positive effects on retention. For example, Nguyen et al. (2023) examined state school finance reforms (SFRs) as potential mechanisms to improve teacher salary, turnover, and job satisfaction by using nationally representative data from 2000 to 2016. They found that SFRs increased teacher salaries by approximately $4,000 and reduced teacher turnover by an average of three percentage points. However, improvements in both outcome measures took up to a decade to manifest.

20. See Zavelevsky and Lishchinsky, 2020; Carver-Thomas and Darling-Hammond, 2019; and Whitfield et al., 2021.

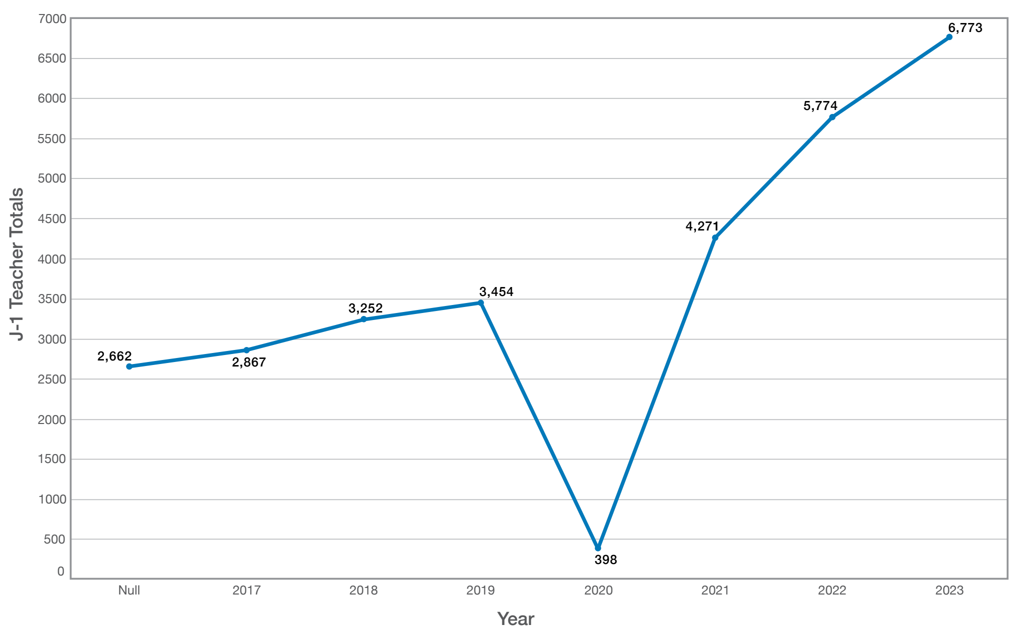
In terms of nonfinancial incentives, the authors noted some evidence in support of retaining early career teachers and ongoing professional development for experienced educators. Evidence supporting mentoring and professional development was generally positive for mentees, but most studies underpinning these conclusions were of limited quality. Moreover, the more rigorous studies failed to demonstrate positive effects consistently. Similarly, there was limited evidence on the effectiveness of specific induction programs in retaining new teachers. The few studies in this area tended to be methodologically weak and reported mixed or inconclusive findings. Even the more robust studies show little or no significant impact. A key challenge is the highly variable and multifaceted nature of these interventions, which makes it difficult to pinpoint, measure, and understand the specific components of induction programs. It is often unclear whether observed effects result from induction alone or a combination of factors. Additionally, some studies measured "intention to remain" in the profession rather than actual attrition rates, further complicating conclusions. These issues highlight the need for more carefully designed and evaluated induction programs in future research.

> "The evidence suggests that for financial incentives to work, they have to be large enough to compensate for the challenges of working in less desirable schools and areas, or to compensate for the salary that teachers would receive if they had been in comparable profession. This is especially so for shortage subject teachers like math and science where graduates from these subjects tend to command a higher salary in the labour market."
>
> (See et al., 2020, p. 25)

Visa Exchange Programs. While alternative TPPs represent a general approach to decreasing entry barriers to the profession, a short-term strategy to increase the teacher supply in specific subject areas, such as mathematics, has been visa exchange programs. In addition to generating headlines, this strategy is surprisingly more common than one might expect. In 2013, the federal government sought public comments regarding proposed regulation changes to the Exchange Visitor Program's teacher category rules due to its excessive use to fill labor needs in U.S. public schools. Currently, visa exchange programs allow teachers from other countries to teach in accredited public or private K–12 schools under a J-1 Exchange Visitor Program for up to five years.

According to available data, the number of foreign teachers brought in through the J-1 Exchange Visitor Program has significantly increased in recent years, particularly in states facing teacher shortages (see Figure 7). Between 2016 and 2023, the **number of J-1 teachers has increased by over 150%**. In 2023, the number of J-1 teachers reached 6,773. North Carolina and Florida hosted the largest number of J-1 teachers, with over 1,000 J-1 teachers each. At the beginning of the 2023-24 school year, North Carolina reported 5,095 unfilled teaching positions. Considering the use of J-1 teachers allowed North Carolina schools to fill open positions, and J-1 teachers during that time filled over 17% of open positions. Additional government data on the subject areas taught by J-1 teachers is unavailable, but state-specific news reports suggest that mathematics is among the top, together with science and special education.

Figure 7: Number of J-1 Visa Teachers in U.S. Public Schools



Research on the effectiveness of J-1 teachers in terms of teacher knowledge and student achievement is needed. In addition, the Department of Labor regulations that typically cover foreign guest workers exempt J-1 teachers, allowing school districts to hire them without pre-approval or labor certification and avoid paying federal FICA taxes. As such, this popular short-term strategy remains problematic due to its unknown efficacy and the financial incentives that may promote hiring temporary labor over local talent.

CONSIDERATIONS FOR IMPROVING THE AVAILABILITY OF QUALIFIED MATHEMATICS TEACHERS:

1. Consider establishing standardized data collection protocols that states can implement to collect teacher workforce data in a systematic and centralized fashion, creating comparable metrics across states that can be aggregated to determine local and national trends at the level of specificity needed to devise policies for addressing supply and demand issues. This would require state education agencies, local education agencies, and teacher preparation programs to collaborate on collecting state/district/school information on teacher demographics, experience, qualification/certification, vacancies, and attrition/mobility that can be disaggregated at the certificate level. States with robust data collection efforts exist and have public dashboards available to guide others—see Colorado, for example. For detailed data collection suggestions, please review the recommendations by NCTQ and ExcelinEd.

2. Consider conducting additional research on the various pathways into the teaching profession based on TPP type and their respective requirements and the extent to which these features differentially affect key proximal (e.g., TPP retention and completion rates), intermediate (e.g., certification exam pass rates, job placements, first-year retention), and distal (e.g., retention, satisfaction, effectiveness, student outcomes) outcomes. Of high interest are the more recent alternative TPPs that offer teaching pathways that place students in paid positions almost immediately, offering a mixture of district- and IHE-based teacher preparation courses. While these alternative pathways have the potential to address several entry barriers into the profession (e.g., alleviating financial concerns associated with the cost of TPPs, gaining job experience, finding a suitable job placement), more research is needed to substantiate desired outcomes, including teacher effectiveness, student outcomes, and retention. Noteworthy iterations of these alternative pathways begin in high school, offering career and technical education (CTE) programs that offer work-based learning experiences in education that can lead to certificates, apprenticeships, and career advancements (e.g., from paraprofessional to teacher). State leaders interested in teacher apprenticeship programs can review and adapt existing policy documents, such as this model provided by ExcelinEd.

3. Consider conducting additional research and gathering longitudinal data to aid policymakers in understanding which incentives are most effective in the long run. Current evidence indicates that financial incentives can serve as an effective short-term strategy for recruiting and retaining teachers and may have the potential for reducing pre-retirement attrition if sustained in a targeted manner. Of interest are packaged approaches that combine financial and nonfinancial incentives, such as paid parental leave and flexible benefit-eligible schedules.

4. Consider offering sustained financial incentives systematically implemented through an incentive program that acknowledges and rewards experienced, high-performing teachers by granting them a special designation on their teaching license or other career-ladder advancements. These teachers can advance but remain in the classroom, take on additional responsibilities, such as coaching and mentoring new teachers, and earn higher pay. States could offer additional financial incentives to encourage these advanced teachers to work in high-need districts. More information on teacher incentive programs can be found here.

5. With recent reduction-in-force (RIF) mandates issued across numerous governmental agencies, review a common RIF approach in education, which is based on the length of time an educator has worked in a district or if they have tenure status. This practice is often called "last in, first out." As such, those hired most recently are the first to be laid off when RIF is necessary. As suggested by several policy organizations, a performance-based RIF helps ensure that districts retain the most effective and diverse teachers. We suggest that state leaders review and adapt existing policy documents, such as the model provided by ExcelinEd.

CONDITION #2: DO MATHEMATICS TEACHERS DEMONSTRATE THE KNOWLEDGE, SKILLS, AND PRACTICES TO DELIVER EFFECTIVE MATHEMATICS INSTRUCTION?

Certification alone does not guarantee that a mathematics teacher has acquired important knowledge, skills, and practices. This highlights the necessity of both pre-service and in-service training, as well as the role that research plays in informing teaching practices. Educators require guidance in navigating the "math wars" and additional support to close the research-to-practice gap.

The importance of teachers in helping students learn mathematics is clear. Research shows that variations in students' mathematics achievement can be linked to differences in teachers' instructional practices. For example, some researchers estimate that differences in teachers' instruction can explain around 12-14% of the total differences in students' mathematics achievement gains over an elementary school year.[21] The impact of a teacher's mathematics instruction on student achievement is amplified when students have effective or ineffective teachers over multiple years,[22] underscoring the importance of understanding the extent to which all students have access to qualified mathematics teachers.

What do we mean when we refer to teachers as qualified or effective? This topic sparks debate among school leaders, policymakers, and researchers, posing a significant challenge in assessing whether students have access to qualified teachers. For this issue brief, including the previous section, we have utilized "certified" as a proxy for "qualified." However, the content of the training and assessments required for certification varies significantly across states and teacher training programs. Therefore, we recognize that a teacher's certification status is an imperfect measure of being "qualified" to deliver effective instruction. We believe that a teacher's certification status is an important, even necessary, condition for being considered a qualified teacher, but it is certainly not a sufficient condition. Nonetheless, it serves as a valuable indicator of a teacher's successful completion of relevant coursework and passing an assessment designed to evaluate important knowledge and skills before entering the classroom.

The bar for entering the teaching profession, and for teaching mathematics in particular, varies between states and between teacher training programs. Most teachers become certified to teach after completing a traditional four-year

21. Gordon, Kane, & Staiger, 2006.

22. Rivkin, Hanushek, & Kain, 2005; Sanders & Rivers, 1996.

undergraduate degree from a teacher preparation program at a degree-granting university. These certifications typically come with hours of practicum, courses on pedagogical methods, and student teaching under the guidance of an experienced teacher. While licensure requirements vary by state, candidates typically are required to pass a competency exam with a minimum score to indicate their proficiency with child development, behavior management strategies, and content knowledge for the subject they will be teaching. Certification requirements are also specific to the content and grade level taught: teachers of elementary education often have a general elementary education certification, while teachers of high school calculus typically must pass an exam focusing on those specific mathematics skills. As such, even within a single subject like mathematics, we expect to find a range of competencies teachers are expected to demonstrate to be certified and, therefore, qualified to teach a specific subject and grade. For example, students preparing to become elementary school teachers may only receive a cursory introduction to teaching mathematics, and the minimum mathematical competency threshold that prospective teachers need to pass is lower when the certification is not mathematics-specific. Typically, mathematics teachers in higher grades need to demonstrate specific knowledge of their chosen area of mathematical expertise, but even this requirement varies between states.

Other teachers become certified to teach through alternative teacher preparation programs, such as Teach For America. We can expect a great deal of variation in the materials, pedagogical styles, and content knowledge that prospective teachers learn within their given preparation programs. For example, Teach For America requires its Corps Members, who hold no prior teaching experience, to attend a multi-week summer course on teaching fundamentals, with very little instructional time given specifically to mathematics teaching, and from there places its teachers in classrooms. This is just one example of a well-known alternative certification program, but it points to larger issues related to measuring teacher quality through certification.

Based on available data from the National Teacher and Principal Survey for school years 2015-16, 2017-18, and 2020-21, the percentage of certified teachers has remained relatively stable, with the last survey indicating that 93% of public school teachers were certified for their position in the the state where they were teaching. However, when disaggregated by years of teaching experience, novice teachers with three years or less of experience showed a markedly different trend. **The percentage of certified novice teachers has declined consistently**, dropping from 82% in 2015-16 to 75% in 2020-21. States with the highest number of noncertified teachers were the District of Columbia

(27.7%), Arizona (15.4%), and Louisiana (13.7%). Available data on mathematics combined with computer science indicate that 60.4% of middle school Math or mathematics teachers and 81.9% of high school mathematics teachers were certified in 2017-18. The percentage of teachers with both a degree and certification in mathematics was 24.4% for middle schools and 58.1% for high schools.

It is insufficient to assume that hiring certified mathematics teachers for open positions guarantees effective mathematics instruction for all students. Teachers can only be as effective as their training during teacher preparation and on-the-job professional development, which, in turn, depends on how well this content aligns with established knowledge about what, how, and when to teach specific mathematical concepts and procedures. Ideally, established knowledge should be informed by rigorous research that, over time, has identified how students learn across their developmental span and determined which mathematical content and pedagogy should be used accordingly. Subsequently, professional organizations should support disseminating these findings in ways that are accessible to practitioners.

The extent to which certified/qualified mathematics teachers are "effective" thus represents a complex question that would require us to examine how the research literature defines effective mathematics teaching, its outcomes, for whom, and under what conditions. Any questions about teacher effectiveness also entail value judgments about the types of outcomes that are important in determining effectiveness, such as student performance on state tests, equity of outcomes among students in the class, and so forth. Short of a separate issue brief, we can merely outline some general challenges that merit further exploration. One challenge relates to the knowledge, skills, and practice we expect mathematics teachers to demonstrate. The second challenge pertains to the systems we use to evaluate a teacher's effectiveness based on relevant job domains (e.g., instructional planning, pedagogy, teaching, classroom management), student outcomes, or a combination thereof. Consequently, we identify two challenges to ensuring that all students have access to qualified mathematics teachers who demonstrate the knowledge, skills, and practices to deliver effective instruction for all students:

1. **Establish consensus on the mathematical content knowledge, skills, and practices needed for effective instruction.**

2. **Establish agreement on the assessment of mathematics teacher effectiveness.**

Challenge #1: Consensus on what mathematical knowledge, skills, and practices teachers should possess is needed to bridge the research-to-practice gap.

The most recent consensus panel on mathematics was held in 2008, making it difficult to translate advancements in mathematics research from the past decade and beyond into practice.

In mathematics education research, teacher content knowledge has been studied extensively, going back to Shulman's definition[23] of pedagogical content knowledge, which refers to the knowledge teachers use to translate particular subject matter into lessons for students. Two central components are knowledge of instructional strategies and representations and knowledge of students' conceptions and misconceptions. Building on Shulman's definition, Ball and colleagues[24] introduced mathematical content knowledge for teaching (MKT), which included (a) common content knowledge (i.e., mathematical knowledge and skills used in settings other than teaching), (b) specialized content knowledge (i.e., mathematical knowledge and skills unique to teaching mathematics), and (c) horizon content knowledge (i.e., an awareness of how distinct mathematical topics are related to each other). However, the extent to which these subdomains are separate and distinct, and MKT's overall importance for teaching and student learning, remains a subject of debate.[25]

A review by the 2008 National Mathematics Advisory Panel examined the relationship between teachers' mathematical knowledge and student achievement, with a focus on teacher certification, mathematics coursework, and direct knowledge assessments. While results were mixed, they generally confirmed the importance of teacher content knowledge. However, reliance on proxies like certification and coursework did not clarify the specific mathematical knowledge and skills needed for effective teaching, especially in elementary and middle school. The panel noted direct assessments of teachers' mathematics knowledge showed the strongest connection to student achievement.

Research on U.S. teachers' mathematical content knowledge has accumulated evidence indicating that many elementary and middle school teachers have limited depth in mathematical content knowledge,

23. See Shulman, 1986.

24. See Ball et al., 2008.

25. e.g., Kersting et al., 2012.

especially compared to teachers in countries with high-performing students on international assessments like TIMSS (Trends in International Mathematics and Science Study) and PISA (Programme for International Student Assessment).[26] For example, several studies[27] examined elementary and middle school teachers' understanding of key mathematical concepts. One study found that teachers had limited knowledge of fraction arithmetic, particularly fraction division, with more experienced teachers demonstrating deeper understanding. Another study revealed that teachers' use of proportional reasoning varied, with those who reasoned proportionally performing better on ratio-related assessments, highlighting the need to focus on proportional reasoning in teacher education. A third study demonstrated that teachers' success in solving fraction word problems was influenced by their problem-solving strategies rather than their backgrounds, highlighting the importance of learning informal strategies and visual representations. Lastly, research on fraction magnitude estimation found that teachers' accuracy was limited, especially in fraction division, and was influenced by their credentials and the grade level at which they taught. Overall, these studies highlight the need for more rigorous mathematical preparation and targeted support for teachers to enhance their conceptual understanding.

Research on mathematics teachers also reveals that elementary school teachers are more likely to report higher levels of negative feelings and anxiety toward teaching mathematics, coupled with low self-efficacy toward their ability to teach mathematics effectively.[28] Multiple studies have shown that teachers pass down these attitudes and beliefs to their students, which can negatively affect student achievement.[29] When students have math anxiety, it affects their working memory and can lead to avoidance of math.[30] Pre-service teachers who have had math anxiety in the past have stated concerns about their ability to teach new concepts and strategies.[31] Comparatively, teachers with high self-efficacy in teaching mathematics are more likely to be willing to try new or different teaching approaches, and are willing to use multiple teaching strategies.[32]

26. See Darling-Hammond, 2000; Stevenson et al., 1993, Ginsburg et al., 2005; Ma, 1999; Schmidt et al., 2007; Ball & Bass, 2000.

27. Copur-Gencturk, 2022; Copur-Gencturk, Baek, & Doleck, 2023; Copur-Gencturk, & Doleck, 2021; Copur-Gencturk & Ölmez, 2022.

28. Knaus, 2017; Novak & Tassell, 2017.

29. Chang, 2015; Gonzalez-DeHass et al., 2017.

30. Ramirez et al., 2016.

31. Stoehr, 2017.

32. Chang, 2015.

Efforts to support teachers' mathematical content knowledge and related student achievement are predicated on a clear understanding of the mathematical knowledge and skills students should acquire throughout their PreK-12 mathematics instruction. Based on their 2008 review of high-quality research studies, the National Mathematics Advisory Panel emphasized the importance of (a) students establishing a strong foundation early on, (b) teachers treating conceptual understanding, procedural fluency, and automatic recall of facts as mutually reinforcing parts of instruction; and (c) the overall promotion of effort in math achievement (as opposed to inherent talent). Regarding instructional practices, the panel cautioned that available research did not support the use of any particular categorical instructional approaches, such as focusing exclusively on "student-centered" or "teacher-directed" practices. Instead, they noted that findings indicated that "some forms of particular instructional practices can have a positive impact under specified conditions."[33]

The panel made a concerted effort[34] to establish a list of the essential concepts and skills that students should learn as preparation for Algebra coursework, identifying three clusters of concepts and skills that collectively represent the crucial foundation of algebra. Specifically, they recommended that elementary and middle school students become proficient with whole numbers, fractions, and particular aspects of geometry and measurement. They argued that the sequential nature of mathematics dictates the importance of foundational skills for learning algebra and that the continual revisiting of topics year after year without closure should be avoided. The panel further provided benchmarks for these three clusters and the major topics of school algebra that should build on that foundation. They also considered social, motivational, and affective factors that influence student math achievement, recommending the use of research-based interventions that emphasize the importance of effort in learning mathematics, reduce math anxiety, and support the task engagement and self-efficacy of Black and Hispanic students.

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33. See p. 11 of the Final Report by the National Mathematics Advisory Panel (2008).

34. The panel reviewed the skills and concepts listed in (a) the grades 1–8 curricula of the highest-performing countries on TIMSS, (b) National Council of Teachers of Mathematics' "Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence," (c) grades K–8 in the six highest-rated state curriculum frameworks in mathematics, (d) a 2007 ACT survey, and (e) a panel-sponsored survey of 743 teachers of introductory Algebra across the country who were asked what students need to learn to be prepared for success in Algebra.

MAJOR TOPICS OF SCHOOL ALGEBRA AS PRESENTED BY THE 2008 NATIONAL MATHEMATICS ADVISORY PANEL

Symbols and Expressions	• Polynomial expressions • Rational expressions • Arithmetic and finite geometric series
Linear Equations	• Real numbers as points on the number line • Linear equations and their graphs • Solving problems with linear equations • Linear inequalities and their graphs • Graphing and solving systems of simultaneous linear equations
Quadratic Equations	• Factors and factoring of quadratic polynomials with integer coefficients • Completing the square in quadratic expressions • Quadratic formula and factoring of general quadratic polynomials • Using the quadratic formula to solve equations
Functions	• Linear functions • Quadratic functions—word problems involving quadratic functions • Graphs of quadratic functions and completing the square • Polynomial functions (including graphs of basic functions) • Simple nonlinear functions (e.g., square and cube root functions; absolute value; rational functions; step functions) • Rational exponents, radical expressions, and exponential functions • Logarithmic functions • Trigonometric functions • Fitting simple mathematical models to data
Algebra of Polynomials	• Roots and factorization of polynomials • Complex numbers and operations • Fundamental theorem of algebra • Binomial coefficients (and Pascal's Triangle) • Mathematical induction and the binomial theorem
Combinatorics and Finite Probability	• Combinations and permutations, as applications of the binomial theorem and Pascal's Triangle

BENCHMARKS FOR CRITICAL ALGEBRA FOUNDATIONS AS PRESENTED BY THE NATIONAL MATHEMATICS ADVISORY PANEL

Fluency With Whole Numbers

1. By the end of Grade 3, students should be proficient with the addition and subtraction of whole numbers.
2. By the end of Grade 5, students should be proficient with multiplication and division of whole numbers.

Fluency With Fractions

1. By the end of Grade 4, students should be able to identify and represent fractions and decimals, and compare them on a number line or with other common representations of fractions and decimals.
2. By the end of Grade 5, students should be proficient with comparing fractions and decimals and common percents, and with the addition and subtraction of fractions and decimals.
   By the end of Grade 6, students should be proficient with multiplication and division of fractions and decimals.
3. By the end of Grade 6, students should be proficient with all operations involving positive and negative integers.
4. By the end of Grade 7, students should be proficient with all operations involving positive and negative fractions.
5. By the end of Grade 7, students should be able to solve problems involving percent, ratio, and rate and extend this work to proportionality.

Geometry and Measurement

1. By the end of Grade 5, students should be able to solve problems involving perimeter and area of triangles and all quadrilaterals having at least one pair of parallel sides (i.e., trapezoids).
2. By the end of Grade 6, students should be able to analyze the properties of two-dimensional shapes and solve problems involving perimeter and area, and analyze the properties of three-dimensional shapes and solve problems involving surface area and volume.
3. By the end of Grade 7, students should be familiar with the relationship between similar triangles and the concept of the slope of a line.

Concerning practices, researchers in mathematics education have emphasized the significance of teaching practices in shaping students' experiences in school mathematics, which can be categorized into nine broad dimensions described in the literature (in no particular order):

Cognitive demand: The level of thinking required of students to engage with the task[35]

Problem-solving: Grappling with a task for which the solution method is not known in advance[36]

Connections and applications: Opportunities for students to recognize the links among mathematical topics and apply them to other disciplines, real-world contexts, and their own cultural and everyday experiences[37]

Mathematical discourse community: A classroom environment in which students are expected to share mathematical thinking with their peers and the teacher using mathematical language[38]

Explanation and justification: Students provide reasons for their solution strategies and proof for their mathematical conjectures and ideas; teachers ask "how" and "why" questions to promote explanation and justification[39]

Multiple representations: Conceptualizing mathematical ideas in various forms such as pictures, written symbols, oral language, descriptions of real-world situations, and manipulative models[40]

Students' use of mathematical tools: Representing abstract mathematical ideas using appropriate technology and hands-on items such as fraction strips, pattern blocks, counters, base ten blocks, compasses, and rulers[41]

Structure of the lesson: The extent to which a mathematics lesson is logically organized and sequenced, conceptually coherent, and appropriately time-allocated, leading students to a deeper understanding of mathematical concepts[42]

Mathematical accuracy: The extent to which mathematical concepts are presented clearly and accurately throughout the lesson, the extent to which they are free of misconceptions, and a teacher's effectiveness in handling misconceptions that arise[43]

35. Stein et al., 2009.

36. NCTM, 2000.

37. Lipka, 2002; Civil, 2002; Hand, 2012; Boero, Bazzini, & Garuti, 2001.

38. Hufferd-Ackles, Fuson, & Sherin, 2004.

39. NCTM, 2000.

40. Lesh, Post, & Behr, 1987.

41. Moyer, 2001.

42. Hiebert et al., 2005; Weiss et al., 2003.

43. Berry et al., 2013.

Additional research also highlighted the importance of attending to students' social identities.[44] For example, effective teachers select and implement high-floor, low-ceiling tasks that promote reasoning and problem-solving and engage students in meaningful mathematical discussions. Small and whole group discussions provide opportunities for the teacher to help students see the connections among representations while centering the discussions on students' strengths and what they *do* know.

The extent to which these research findings inform practice has remained a longstanding concern, with many practitioners relying on recommendations from national organizations to summarize available research and inform their daily teaching practices. For example, the National Council of Teachers of Mathematics (NCTM) offers numerous resources that provide practitioners with research-based recommendations for teaching standards and practices. NCTM's 2014 Principles to Actions provided mathematics teachers with eight research-based teaching practices: (1) establish mathematics goals to focus learning, (2) implement tasks that promote reasoning and problem-solving, (3) use and connect mathematical representations, (4) facilitate meaningful mathematical discourse, (5) pose purposeful questions, (6) build procedural fluency from conceptual understanding, (7) support productive struggle in learning mathematics, and (8) elicit and use evidence of student thinking.

Given the substantial research on mathematical knowledge, skills, and practices, the historical underperformance and lack of progress of students on national and international mathematics assessments raise questions about an existing research-to-practice gap and the extent to which a research-based consensus on what should be taught and how exists. The recent debate over the science of reading may be a harbinger of math-related controversies to come, as some news reports have suggested another round of "math wars" over the science of math.

In reading, many thought and school leaders identified Lucy Calkins's "Units of Study" as the most effective way of teaching reading to students, based on the idea that students should learn to love to read and that context clues rather than phonemic awareness should be used to help decode new words.[45] Districts invested millions of dollars in materials and professional development to train teachers on this reading system, largely abandoning the teaching of decoding words and

44. Aguirre, Mayfield-Ingram, & Martin, 2013, 2024; Bartell et al., 2017; Jackson & Delaney, 2020; Lambert, 2020.

45. For a detailed description of the fallout over Units of Study, see https://www.theatlantic.com/magazine/archive/2024/12/lucy-calkins-child-literacy-teaching-methodology/680394/.

phonemes. However, recent research has illustrated that the concepts behind the Units of Study were not based on scientific study and that trying to teach beginning readers in such an abstract manner was especially detrimental for students who did not already come from homes with high literacy levels.

In mathematics, a group of interdisciplinary researchers, educational consultants, and university trainers has advocated for a "science of math movement" in response to students' chronic underachievement and concerns that many mathematics teachers continue to employ disproven instructional practices or those not adequately supported by empirical research. In their published writings, they have called into question several common mathematical practices, including some recommended by national organizations, based on concerns of flawed or limited research. They have also questioned certain "myths" about teaching mathematics that suggest (a) conceptual understanding should proceed procedural understanding, (b) teaching step-by-step procedures for solving problems is harmful, (c) inquiry-based learning is the best approach to introduce and teach math, (d) productive struggle is important, (e) growth mindset increases achievement, (f) executive function training is important, and (g) timed assessments cause mathematics anxiety.[46] Critics of this movement have argued that most mathematics researchers do not share these concerns, suggesting that their studies are focused too narrowly on students with disabilities. While we will seek to establish areas of consensus surrounding this controversial topic in a future brief, the larger context and implications are important for understanding the landscape of students' access to qualified mathematics teachers.

The debate over the teaching of mathematics leads inevitably to debate over how teachers should instruct students, how teachers' effectiveness as mathematics teachers should be assessed, and how student performance on mathematics assessments reflects teacher effectiveness. If mathematics research cannot come to a consensus on what and how mathematics should be taught, we cannot expect teachers to be adequately prepared and ready to teach mathematics in ways that can lead to higher mathematics achievement for all students. An ongoing debate among experts that fails to identify areas of consensus and does not lead to more and improved research will leave practitioners in a vacuum, with the potential to further erode the public's confidence in experts and make the teaching and learning

46. See Powell, Hughes, & Peltier, 2022.

of mathematics more susceptible to political or ideological influences. These concerns merit consideration, especially considering that survey data indicate that evidence-based practices in mathematics have not yet reached widespread adoption.[47]

Challenge #2: There are many ways to assess teacher effectiveness, but most assessments are not subject-specific.

Popular assessments of teacher effectiveness report on broad domains of teaching and are not designed to pinpoint the extent to which specific mathematical knowledge, skills, and practices are being implemented.

Teacher effectiveness is a broad and heavily debated topic, with several prominent teacher evaluation methods arising over the past few decades. These attempts to quantify and qualify teacher ability rose to prominence in conjunction with the focus on accountability under the No Child Left Behind Act of 2001 (NCLB) and the increased pressure placed on state and local education agencies to improve student achievement scores. In an attempt to understand challenges related to student performance and achievement gaps, federal mandates required states to devise systematic ways to measure the contribution of individual teachers. In the wake of NCLB, teacher evaluations were explicitly and almost exclusively linked to student achievement on standardized tests. During this time, teacher effectiveness was associated with their students' ability to demonstrate baseline proficiency standards and to meet Adequate Yearly Progress.

The authorization of the Every Student Succeeds Act (2015) ended the federal requirement to include student achievement measures as part of a teacher's evaluation. States were allowed to develop their own systems for teacher evaluation that included multiple measures, including student performance, as part of a larger set of measures of teacher performance. Classroom observation of teachers was and still is an important component of understanding teacher effectiveness, but there is now a focus on multiple observations by different viewers to establish inter-rater reliability and eliminate bias that could influence ratings. These teacher evaluations are meant to be informative in helping school leaders develop professional learning goals with their faculty, provide feedback, and measure growth.

47. Hott et al., 2019; Peltier et al., 2021.

Teacher evaluations are now common and often used to measure teacher effectiveness. Districts collect data on teacher evaluations and frequently include it on their yearly report cards because it can be a component of receiving state funding. Teacher evaluations are also a metric associated with equitable access. In theory, states want to ensure that effective teachers are equally distributed among high- and low-performing schools and high- and low-income areas. Nearly every state in our state-by-state analysis included an indicator for teacher effectiveness.

The fact that a single evaluation method has not proven to be widely applicable and sustainable for a long period of time highlights the difficulty in judging teacher performance. Currently, most states include (a) teacher-level observational measures, (b) value-added models (VAMs), (c) student surveys, and/or (d) student learning objectives (SLOs) as part of their evaluation system. Available data indicate that observational measures are used in the majority of states (71%), followed by SLOs (55%), VAMs (29%), and student surveys (27%).[48] A caveat for many of these frameworks is the limited focus on mathematics instruction. These frameworks are all designed to capture teaching practices generally, not specific to mathematics instruction, and therefore do not offer any specific insight into teachers' pedagogical or content knowledge for teaching mathematics.

One popular observational framework is the Framework for Teaching (FFT), proposed by Charlotte Danielson in the 1990s, which focuses on core competencies for teaching that aim to foster teachers' self-reflection, growth, and improvement, while also providing a means for others to evaluate their teaching. The evaluation is organized into four domains that are not intended to be subject-specific: planning and preparation, learning environments, learning experiences, and principled teaching. Notably, central to the FFT is the idea that teaching is a calling, and its values and practices should be maintained even outside of the classroom. This qualitative evaluation system attempts to measure value-laden aspects of the teaching profession that are not necessarily aimed at improving teachers' efficacy in teaching math, because the measure of effectiveness is teachers' ability to raise student test scores.

The Focused Teacher Evaluation Method, created by researcher Robert Marzano,[49] is a more data- and evidence-driven approach to analyzing teacher performance. This methodology relies heavily on student achievement data to measure teacher growth, with a strong emphasis on instructional practices aligned to standards implementation. The four domains of expertise measured in the teacher evaluation are standards-based planning, standards-based

48. Close et al., 2020.

49. See Marzano & Toth, 2013.

instruction, conditions for learning, and professional responsibilities.

A three-year research project funded by the Gates Foundation, the Measures of Effective Teaching (MET) study,[50] sought to comprehensively understand the validity of different ways of measuring teacher effectiveness. Using classroom observations, student surveys, and student achievement data to assess the effectiveness of 3,000 classroom teachers, researchers concluded that a single measure is insufficient to capture teacher effectiveness. Classroom observations, in particular, are susceptible to bias from the observer, and researchers found that there was more variation between two observers watching the same teacher than between two lessons taught by the same teacher and observed by the same person. This suggests that classroom observations should be taken as only part of a measure of teacher effectiveness.

Researchers have also attempted to evaluate teacher effectiveness by quantifying the contribution of an individual teacher to a student's academic progress. Value-added models (VAMs) use longitudinal student achievement data to isolate the achievement gains associated with an individual school year or teacher. Researchers have proposed several different models for measuring the value added by individual teachers, with covariates accounting for prior student achievement and family characteristics that could influence academic performance. Ultimately, conclusions about the utility of value-added models depend on the availability of data and the underlying assumptions made by researchers about the variables to include in the model.[51]

A further challenge with understanding teacher effectiveness as it relates to equitable math instruction is that most evaluations are meant to be "one-size-fits-all." They are agnostic with regard to subject matter and content expertise and instead focus on practices, skills, and dispositions that carry across all subjects. This is for convenience as well as fairness. We want all teachers to be evaluated using the same metrics. We also do not expect school leaders to have in-depth content or pedagogical knowledge to evaluate every subject in their schools. We do expect them to be able to observe teachers' overall teaching methods, presentation, and skills without regard to specific knowledge of calculus. While schools may disaggregate their teacher effectiveness measures by subject area, it is important to note that the evaluations themselves are rarely or only partially specific to math content.

50. See Kane, et al., 2013.

51. Sass et al., 2014.

TAKEAWAYS:

1. Mathematics teachers must demonstrate the knowledge, skills, and practices for their respective teaching assignments to effectively advance student achievement. Practitioners must also know their students and design instruction to meet their needs.

2. Efforts to assess and advance mathematics teaching require clarity about which knowledge, skills, and practices are supported by rigorous research.

3. Researchers and other mathematics experts play a critical role in synthesizing available research, reaching consensus, and guiding practitioners.

4. Despite major efforts to summarize research about what and how to teach mathematics, including by the National Research Council and the National Mathematics Advisory Panel, the extent to which their conclusions and recommendations have informed teacher preparation, practice, and professional development of mathematics teachers remains unclear.

5. An updated research consensus would support efforts to improve mathematics education, clarify what is known about teaching and learning mathematics, and put any debates under the umbrella of the “math wars” into perspective. Healthy debate about what works, for whom, and under what conditions is part of scientific progress and should not lead to the stifling of inquiry or wholesale questioning of an entire discipline.

Strategies for supporting the implementation of important mathematical knowledge, skills, and practices focus on in-service professional development.

Recent research, which uncovers the most effective components and modalities for transferring knowledge in the classroom, can inform professional development.

Teacher professional development is the most common way to enhance teachers' knowledge and skills, keep up with research, and improve the capabilities of those in the classroom who are not yet qualified to teach mathematics. Researchers have made considerable advancements in recent years in understanding the most effective components and modalities of professional development.

Traditional professional development workshops—those offered on a single day or before the start of the school year—faced criticism when researchers began examining their effectiveness and found that they did not lead to the anticipated application of knowledge and skills in the classroom. Reviews and meta-analyses of professional development uncovered several core components associated with improved teacher and student outcomes. Notable core components of effective professional development include a **specific content focus** (e.g., focus on math teaching strategies for math teachers), **active learning** (e.g., novice teachers observing experts or being observed; interactive feedback), **coherence** (teacher learning should be consistent with teachers' prior knowledge and beliefs; messaging should be consistent across different channels), **collective participation** (among teachers from the same school, grade, or department), and adequate **duration**.[52] In 2022, the Research Partnership for Professional Learning published a research brief[53] based on a larger literature review summarizing key characteristics of effective professional learning design related to professional learning format and focus. In terms of format, they highlighted the need for collaboration, coaching, and follow-up meetings. Regarding focus, they noted pedagogy related to specific instructional practices (rather than general foci) and resources that support teachers with concrete curricular materials (rather than a focus on general principles).

Strategy Example: Instructional coaching as a key modality for implementing sustained, job-embedded professional development.

52. See Desimone, 2009.

53. Hill & Papay, 2022.

Instructional coaching can address all the core features of effective professional development, but requires systematic implementation to work at scale.

One PD modality ideally positioned to address these core components is instructional coaching, which uses a school- or district-based coach to support individual teachers through ongoing job-embedded PD. As such, PD is individualized to the needs of specific teachers and delivered on the job when and where teachers need the support. Moreover, coaching allows PD to occur across the school year in a sustained fashion. Initially, district leaders had to develop their own ways of selecting and training coaches due to a lack of guidance from research. Over time, however, research studies on coaching have identified core features and components of effective coaching, leading to a rise in the adoption of instructional coaching programs. Recent data from the Schools and Staffing Survey and the National Teacher and Principal Survey showed a significant increase in the prevalence of instructional coaching programs in U.S. public schools from 2008 to 2016, with a rise from 33% to 44% of schools implementing these programs. This growth was particularly notable in suburban and town schools and schools with lower poverty concentrations, although schools with high poverty levels consistently had higher rates of adoption.[54]

Several factors have contributed to the popularity of instructional coaching, including (a) research findings showing causal effects of coaching on improved teaching and student achievement, (b) coaching's effectiveness compared to traditional workshop-based professional development, and (c) the expansion of teacher career ladders to offer coaching positions, potentially improving teacher retention.[55] A meta-analysis of 60 coaching studies showed that while coaching can be effective, it is resource-intensive, and its success may vary depending on specific program design features. In terms of cost-benefit, more research is needed as very limited information is presently available, with one study estimating the cost of traditional on-site coaching ranging between $3,300 and $5,200 per teacher.[56] Nonetheless, coaching can positively impact teachers' instructional practices and student achievement, with effects larger than or comparable to other significant interventions, such as adding more veteran teachers, comprehensive school reform, and high-dosage tutoring.

54. See Redding, Tan, & Hunter, 2024.

55. See Kraft et al., 2018; Hunter & Redding, 2023; Donaldson & Johnson, 2011.

56. See Knight, 2012.

In the area of mathematics, research into coaching suggests that well-designed coaching initiatives can result in measurable gains in teaching improvement and student achievement.[57] In a randomized controlled trial, researchers found that mathematics coaches positively affected elementary students' mathematics achievement, particularly after coaches received extensive professional development.[58] A recent study examining the Tennessee Math Coaching Model found that deep and specific pre-lesson planning discussions between coaches and teachers focused on content, pedagogy, and student learning, enhanced teachers' ability to maintain the cognitive demand of high-level mathematics tasks. This is crucial because maintaining such demand encourages student engagement with complex tasks, leading to a better conceptual understanding of mathematics. This is particularly important considering the rigorous college- and career-readiness standards in current educational policies. By analyzing data from multiple improvement cycles, the researchers underscored the importance of pre-lesson planning conferences as a vital opportunity to prepare for instruction. By aligning discussions with the instructional triangle, teachers can incorporate student-centered and conceptually oriented strategies into their teaching. Even limited coaching—just two or three cycles annually—had a positive impact on teaching. Therefore, pre-lesson planning conferences are recognized as a high-leverage coaching practice for enhancing mathematics instruction.

Strategy Example: Targeted, high-quality professional development options.

Professional development can fall on a continuum from adaptive to specified.

Developing different models of PD, particularly those that reach historically difficult-to-reach teachers, such as those in rural contexts, is essential to improving historically underserved students' access to qualified mathematics teachers. PD may also vary with regard to foci, with some programs largely centered on pedagogy, others focused on content, and many others that merge the two. For this example, we focus not on the form or foci of PD but on the level of adaptability of PD that aims to help teachers learn content, apply valued mathematical practices, and promote equitable mathematics teaching.

57. Allen et al., 2011; Biancarosa et al., 2010; Blazar & Kraft, 2015; Bryk et al., 2015; Campbell & Malkus, 2011; Foster & Noyce, 2004; Kraft et al., 2018; Matsumura et al., 2010, 2012, 2013; Neuman & Cunningham, 2009; Powell et al., 2010; Sailors & Price, 2010.

58. Campbell & Malkus, 2011.

One way to classify PD models is to consider where they fall on a continuum from adaptive to specified.[59] Adaptive models typically involve setting learning goals and utilizing resources influenced by the local context, with teachers sharing artifacts of practice such as student work or videos from their classrooms. When teachers focus on learning new pedagogical practices, such as implementing specific mathematical practices or strategies to support more equitable instruction, an adaptive model allows the facilitator and/or teachers to select the goals of the PD and determine how they will work together to learn, implement, and reflect on these new practices. Teachers may want to share artifacts from their classroom to anchor adaptive professional learning sessions with activities based on general guidelines that address the group's needs and interests in reaching the identified goals. Individual coaching sessions are typically adaptive, and the teacher and/or coach may identify goals for particular sessions based on the needs of the particular teacher.

Examples of adaptive mathematics PD models are book or video clubs[60] and lesson studies.[61] A video club is adaptive because it typically consists of a group of teachers at one school who want to study their teaching practice by watching videotapes of their own teaching and analyzing student thinking during club meetings. They engage in discussions about student thinking and ways to support student learning. Another common form of adaptive PD is lesson study. Lesson study is a hybrid PD model, falling on the adaptive side of the continuum but more toward the middle. Lesson study is a model of PD that has been increasingly practiced in the U.S. but originated in Japan. Lesson study is adaptive because it typically consists of a group of teachers from one school. These teachers collaboratively determine the focus of the research question and topic of a collaboratively designed lesson, usually with a facilitator leading the group.

Some recent research has shown that participation in lesson studies designed for U.S. teachers, especially with a predetermined mathematics focus, resulted in a positive impact on teachers' knowledge and pedagogical content knowledge[62] and improvements in student learning.[63] Additional studies included evaluations of the impact on the norms and values of PD in general.[64] Results indicated that

59. See Borko et al., 2011; Koellner & Jacobs, 2015.

60. Sherin & Van Es, 2009.

61. E.g., Murata et al., 2012.

62. Lewis & Perry, 2017; Ní Shúilleabháin, 2016.

63. See Lewis & Perry, 2017; Perry & Lewis, 2009; Perry et al., 2006.

64. Lewis & Perry, 2015.

teachers who participated in lesson studies became more collaborative, sought more critique and feedback from colleagues, and shifted their focus to emphasize content, student thinking, and the ways that students interact with the content.[65] This is a nice example of a hybrid model of PD landing in the middle of the continuum as they used an adaptive structure, identified goals, specified content, and pedagogy, and showed an impact on teachers and students.

On the other end of the continuum, specified models of PD typically incorporate highly specified goals and materials, sometimes published, that determine teacher learning goals in advance. In specified PD, the designers of the PD typically focus on a specific mathematical content domain or specific mathematical pedagogy. The designers include artifacts in the materials (e.g., video clips, student work, books); these artifacts are prepackaged, selected from other teachers' classrooms in an attempt to systematically support the learning of objectives. One commonly known example of a specified PD is called cognitively guided instruction. This PD intends to help teachers recognize how children conceptualize arithmetic concepts. Teachers learn frameworks that have been developed to classify students' different ways of thinking and categorize arithmetic problem types. This is a specified PD because these frameworks are at the heart of the PD. Teachers receive a book or handbook that is a specified guide. During PD sessions, teachers collaboratively learn about student thinking and the problem types as they watch videotapes from other teachers' classrooms and analyze the student thinking using these frameworks. The goals and intentions of this type of PD are clearly articulated and can be easily assessed at different points throughout the PD.

The adaptive-specified continuum construct helps educators consider how to structure and target professional learning experiences in alignment with particular goals and contexts. A coach might begin their work with teachers by encouraging them to examine specific, research-based practices of their choice via books, articles, or webinars. This is highly adaptive because the teachers can adapt their learning to their needs and interests. However, if the teachers later coalesce with a desire to focus on developing their skill in posing purposeful questions, the coach might then plan a more specified approach to supporting the teachers with planned-out resources and a sequence of activities particular to that aim. The value in viewing PD along

65. Lewis & Perry, 2015; Martensson & Hansson, 2018.

this continuum is to provide language and a frame around which to consider the most productive approaches to supporting teachers in improving their skills and practice.

Identifying the position of a PD model on the continuum can clarify choices regarding the targeted goals of teacher learning. This continuum serves as a broad framework to help teachers, coaches, and district leaders understand what to expect when teachers engage in specific models. For example, if a new math curriculum or content unfamiliar to teachers is being implemented in the school, a specified PD focused on that content might prove beneficial. If teachers are already acquainted with the content, creating an adaptive learning community in which they can plan, implement lessons, and reflect collaboratively is the optimal choice. Although PD models vary in their placements on the continuum, they frequently share similar design features recognized as effective for teacher learning.

CONSIDERATIONS FOR IMPORTANT KNOWLEDGE, SKILLS, AND PRACTICES TEACHING MATHEMATICS.

1. Consider more centralized, robust data collection on the various teacher preparation pathways and their effectiveness. At a minimum, we could collect data to compare the certification requirements for each pathway into teaching, and in particular, the requirements for teaching mathematics within each grade band.

2. Consider using statewide data to identify mathematics teachers who are associated with improvement in student math achievement over the course of multiple assessment cycles. Analyze the paths that lead to successful instruction (e.g., preparation program, degree field, completed professional development opportunities) to use as recommended best practices for policy and practice decisions.

3. Consider using content assessment for teaching mathematics, even at the elementary level, with the expectation that teachers understand the scope and sequence of mathematics topics, both above and below their grade level. Assessment results could be used to inform targeted PD opportunities.

4. Consider implementing using statewide mathematics training for teachers and administrators with ongoing, job-embedded mathematics professional development and support for teachers via mathematics coaches.

5. Consider providing professional learning opportunities for teachers to enhance their content-specific expertise in teaching mathematics (i.e., a robust understanding of the mathematics taught in school and pedagogical content knowledge).

CONDITION #3: DO MATHEMATICS TEACHERS USE HIGH-QUALITY INSTRUCTIONAL MATERIALS?

Instructional materials should be rigorous, cohesive, aligned to state standards, and supported with teacher professional development.

It is not enough to assume that students will learn math provided they have access to a qualified mathematics teacher. It is also necessary for teachers to use high-quality instructional materials to implement the curriculum. Even a qualified teacher may struggle to address students' needs without aligned instructional materials that are also designed to support different types of students. The standards themselves do not dictate the content of daily lessons but merely provide learning goals for teachers and students to reach by the end of the year. Instructional materials form a bridge between state standards and classroom instruction, guiding teachers as they plan and enact their lessons. We know that the content, particularly the scope and sequence of instructional materials, factors into teacher decision-making around lesson planning.[66] In the vacuum between the intended standards and daily classroom instruction, teachers call upon instructional materials to decide what and how to teach. If these materials are lacking in quality, rigor, alignment, coherence, or adaptability, teachers will look elsewhere to find suitable materials, a process that can be time-consuming and ineffective. Recent American Instructional Resources Survey data collected by the RAND Corporation found that a majority of teachers spend at least a few hours each week searching for additional instructional materials to supplement those provided by their schools.[67] Over half of all teachers surveyed also report using instructional materials they create themselves because they find the quality, alignment, or rigor of their school's adopted materials insufficient.

Instructional materials are not a static artifact; they are a dynamic component of instruction that influences and is influenced by teachers and students. Teachers make decisions about how to use instructional materials based on factors such as usability, perceived alignment to standards, and ability to meet diverse student needs. Based on their evaluations of instructional materials, teachers may choose to implement them with fidelity, attempting to adhere to them as closely as possible. They may ignore the recommendations in the materials entirely, seeking alternate sources. Or they may adapt the written

66. Opfer et al., 2016.

67. Doan et al., 2022.

materials, choosing which parts to include and exclude based on their students' needs and supplementing with outside resources as they see fit. In almost all cases, teachers mediate the introduction of instructional materials to students, making decisions about which materials students will use and what students will be doing with the materials. Therefore, the use of instructional materials is intertwined with a teacher's knowledge, skills, and expertise in the subject matter, thus representing a key component for understanding the landscape around equitable math instruction.

Historically, there have been relatively few rigorously designed studies comparing the effectiveness of various mathematics curricula. However, the scant research that does exist indicates that the selection of a particular set of materials can make a difference in student achievement. Researchers examining the adoption of popular mathematics textbooks in Indiana and Florida found differences in student achievement associated with the choice of one set of materials over another.[68] These achievement effects even persisted into the next school year. Additionally, evidence shows that the achievement effects associated with instructional materials selection are even larger among students in low-income schools. Because there is only a marginal price difference when choosing between sets of instructional materials, educational agencies can choose a more effective set of materials to support instructional practices at little to no extra cost to a district.[69] This is, of course, assuming that researchers have valid data on the effectiveness of different sets of materials, and that this information is made available to educational agencies as they make decisions about new materials to adopt.

High-quality instructional materials are essential for ensuring students have access to effective teaching. However, challenges arise in getting these high-quality materials into classrooms and ensuring they are used effectively by teachers. The key challenges we identify are:

1. **Forming consensus on curricular scope and sequence in mathematics**
2. **Negotiating the curriculum adoption process**
3. **Measuring alignment of materials to standards**
4. **Overcoming limitations of the local evaluation and adoption process**

68. Several rigorous experimental and quasi-experimental designs have shown that choosing one math curriculum over another can be associated with differences in student achievement (Agodini & Harris, 2010; Bhatt & Koedel, 2012; Bhatt, Koedel, & Lehmann, 2013; Polikoff, 2017).

69. Boser, Chingos, & Straus, 2015.

After discussing each of these challenges, we present some promising strategies in the instructional materials space and highlight some particularly effective ones that could successfully be adopted more widely.

Challenge #1: We lack consensus on the appropriate scope and sequence of mathematics curricula.

This leads to variations in the mathematics content that students have the opportunity to learn.

At a systems level, ensuring all students have access to high-quality instructional materials is challenging because we lack consensus on the best content and sequencing of students' mathematical learning. While experts agree that understanding place value in whole numbers should come before decimals, and addition should come before multiplication, there is less agreement over the specific algorithms students should learn or the sequencing of concepts like geometry and metric measurement.

This means that the content of mathematics curricula can vary significantly—a student in one state may learn how to do double-digit multiplication in third grade, while a student in another state may not encounter these algorithms until fourth grade.

Challenge #2: The curriculum adoption process is heavily influenced by publishers trying to sell to the largest markets.

This means that states with the largest student populations influence the content of instructional materials.

Without a standardized consensus on the best scope and sequence of math curricula, publishers align their materials to available sets of standards. States with the largest populations of students (e.g., California, Texas, Florida) represent the largest markets for instructional materials, so publishers make a concerted effort to align their materials to these states' standards. As a result, the content of national curriculum materials can be influenced by, or even recycled from, materials written specifically for these states and their standards.

This can be problematic when the complexity, scope, and sequence of math standards vary widely between states. Additionally, partisan interest groups have a significant influence on the curriculum materials adopted in certain states, such as Florida and Texas. In these states, the committees that review publisher submissions of curriculum materials are often composed of members with a particular interest in keeping certain topics out of the classroom or pushing an agenda through curriculum materials. For example, book bans in these and other states have resulted in any references to concepts like critical race theory, gender and sexual identity, and climate change being removed from the curriculum entirely. In a recent math textbook adoption year, Florida rejected 54 out of 132 submitted textbooks due to the inclusion of references to a wide range of topics, including anything interpreted as critical race theory, social-emotional learning, or the Common Core.

Challenge #3: True "alignment" to new standards is questionable, especially when materials are published quickly to meet new adoption deadlines.

Teachers and school leaders are also skeptical of publisher claims of alignment.

Another challenge in the publishing industry is the timeline—publishers typically work to produce materials "aligned" with new standards as quickly as possible so they have a chance to be approved and purchased by educational agencies seeking new resources to support instruction. This timeline creates a tight turnaround for publishers wanting to market their instructional materials as aligned with the latest standards, even though there may not be sufficient time for a meaningful overhaul. The early rounds of materials that were supposedly aligned with the Common Core math standards were not, in fact, well aligned with the rigor and conceptual knowledge required by those standards. One common method for analyzing the alignment of curriculum materials to standards is a tool called the Surveys of Enacted Curriculum, or SEC.

In a content analysis of the most popular math textbooks published to align with the Common Core, researchers found that the best-aligned textbooks were only 28% to 40% in alignment with the standards. Even several years later, researchers found that alignment to the standards was not much improved: while alignment ranged from 36% to 60%, the average alignment between math materials and the Common Core Standards in math was only 51%.[70] While curriculum materials may fail to cover the topics and cognitive demand intended by the standards, they also contain extraneous material not aligned to the standards, covering skills that students at that grade level are not expected to learn. In their analysis of Common Core-aligned math materials adopted in California, a research team using the SEC found that 40% to 64% of the content in the materials was not included in the

70. Polikoff et al., 2021.

Common Core Standards for that grade level.

Teachers and district leaders also feel that publisher materials are poorly aligned and inadequate in supporting teachers as they implement new standards. This raises questions about how effectively the instructional materials can assist teachers in their pedagogical practices.

Challenge #4: Local education agencies are hampered by short timelines and limited resources when they consider new materials for adoption.

Evaluating and piloting new materials takes time away from classroom instruction.

There are also local challenges related to using curriculum materials as a key factor in educational equity. One difficulty is the time required for a school or district to assess a set of curriculum materials, or potentially several sets simultaneously. In many states, individual districts are tasked with selecting new curriculum materials, and this process typically involves an evaluation cycle with teacher input and the piloting of new materials. This effort demands a significant investment of resources that can divert attention from classroom instruction. Moreover, teachers and school leaders express that they lack dependable information to assess the quality of the materials being considered. They depend on recommendations or approvals from state boards of education and on publishers' claims of alignment and quality, but they are eager for an impartial evaluation.

The challenges described above create barriers to accessing high-quality mathematics instructional materials. Instructional materials do not exist in isolation; they are artifacts interpreted and utilized by teachers, who are influenced by their beliefs, mathematical knowledge, pedagogical experiences, and the dynamics of the classroom at any moment. Even qualified teachers may struggle to meet students' needs if they lack adequate instructional materials that align with the standards. Conversely, an under-qualified teacher may fail to address student needs, even when equipped with the most rigorous and well-aligned instructional materials.

TAKEAWAYS:

1. Instructional materials serve as a bridge between the language of standards and the teaching methods employed by educators. Teachers rely on these materials to make decisions regarding what, when, and how to teach various topics.

2. Assessing the quality and alignment of instructional materials can be challenging, and local education agencies often lack the resources for thorough analyses of multiple materials being considered for adoption.

3. Even an effective mathematics teacher may struggle to align their instruction with the standards without the backing of well-aligned, high-quality, and rigorous instructional materials.

Strategies to support access to high-quality instructional materials focus on criteria that local education agencies can adopt.

Local education agencies need access to reliable and independent criteria for evaluating materials, and we need agreement on the mathematics content that students should learn.

The challenges outlined above are barriers to ensuring that all students have access to high-quality math materials. However, we observe innovative solutions emerging from state boards of education, mathematics experts, and independent companies. Overall, access to high-quality instructional materials can improve if districts have reliable criteria and processes for adopting new materials and if we reach a consensus on the content students should learn.

Strategy Example: Establish a clear agreement on the mathematics content that students should learn.

This will eliminate variability in the content of instructional materials and ensure equal opportunity for students to learn the same math skills.

One way to achieve equitable access to high-quality instructional materials is by reaching a consensus on the mathematical content students should learn. This ensures that all students in a particular grade can be expected to learn the same material, thereby minimizing differences in content and sequencing within instructional resources. Currently, individual states are responsible for adopting their own math standards. While there may be considerable overlap between states, variability still exists. These variations in the scope and sequence of learning standards lead to differences in instructional materials and the exposure students receive to specific math strands. For instance, one state might introduce geometric solids in fourth grade, while another may not introduce them until fifth grade. These differences become obvious when students move between states and encounter a fragmented learning progression and when standardized tests like the NAEP highlight specific content areas in which students in particular states struggle.

The adoption of the Common Core Standards was the closest we have come to achieving consistency in curriculum expectations for all students. Beginning in the early 2010s, many states chose to adopt the Common Core Standards or a variation of them. This marked the first time that an almost national set of math standards was available. Theoretically, widespread adoption of rigorous standards should allow publishers to align their materials, fostering consistency and coherence in the quality of resources used across the country. If students in rural Oklahoma are using materials aligned to the same standards as students in Greenwich, Connecticut, there is a greater opportunity for equitable math instruction.

Strategy Example: Use educational technology to innovate with teaching.

Open educational resources can provide free, high-quality, and aligned materials to support instruction.

Another innovation comes from educational technology. Traditional textbooks are no longer the dominant force they once were, as numerous open-access and independent educational technology companies produce materials accessible to anyone with an internet connection. This technology, which better addresses the needs of teachers and students, is disrupting the traditional educational publishing industry. The accessibility and popularity of features such as digital textbooks and interactive learning experiences are notable. Even traditional publishers are incorporating features like adaptive assessments and interactive lessons into their instructional materials as they try to compete with independent curriculum developers. In RAND's recent survey on instructional materials, 22 % of teachers reported using open educational resources as part of their instructional materials.

Strategy Example: Use independent evaluations of instructional materials to provide useful information for local education agencies.

These rigorous evaluations offer impartial insight into the alignment and usability of new instructional materials.

Another promising innovation is the emergence of independent groups that empirically assess the quality, rigor, and usability of curriculum materials. The most notable and well-known of these organizations is EdReports, which employs a rigorous methodology to evaluate the alignment and usability of instructional materials. EdReports reviews can be instrumental in assisting schools and districts in evaluating new materials before adoption. The National Center for Education Statistics also examines research on topics such as curriculum materials and publishes the findings in the What Works Clearinghouse. This information enables school leaders to assess the quality of research on materials that may be under consideration, although it is not a comprehensive resource. It reviews the already limited number of studies on the efficacy of individual curriculum programs.

Various websites offer evaluation criteria and rubrics for schools to use when selecting new materials for adoption. The EQuIP website provides tools and a rubric that teachers can utilize to assess the alignment of instructional materials with the Next Generation Science Standards. Another valuable resource for evaluating instructional materials is the IMET (Instructional Materials Evaluation Tool). Although this toolkit was created with the adoption of the Common Core Standards in mind, its resources remain useful as they focus on addressing the inherent biases found in instructional materials. From the perspective that "instructional materials play a role in disrupting racist systems that continue to devalue, ignore, and fail to recognize the inherent brilliance of Black students, students who are English learners, and others," the IMET can offer guidance on better supporting historically underserved students through instructional materials.

The Council of Chief State School Officers (CCSSO) is a reputable resource that presents a roadmap and links for states to consult when evaluating curriculum materials. The CCSSO acknowledges that the implementation of high-quality instructional materials relies on teachers' understanding. Hence, they offer additional recommendations for states to assist teachers during the implementation of new materials. These professional development resources have been vetted and are aligned to enhance the pedagogical strategies and content knowledge needed to implement new standards and instructional materials.

Strategy Example: State educational agencies can provide guidance and incentives to local education agencies as they make decisions about materials adoptions.

Informational toolkits can reduce the burden on local education agencies, and financial incentives can encourage districts to adopt high-quality materials.

State and federal policies, amplified before and during the pandemic, have provided additional focus on the need for high-quality instructional materials. In 2015, Congress passed the Every Student Succeeds Act, or ESSA, which provides incentives for schools to use programs with proven evidence of effectiveness. It was followed by the Elementary and Secondary School Emergency Relief of 2020 (ESSER) Fund to support instruction as students and teachers returned to schools under unprecedented circumstances and the need to recover the equivalent of years of learning loss. Evidence for ESSA is a website that curates a database on programs with evidence of effectiveness, and the Professional Learning Partner Guide provides a searchable database of vetted professional development that supports the implementation of high-quality instructional materials. Under these two acts, schools that use effective materials are eligible to receive additional funding or are incentivized to use materials that have been rated highly effective.

The CCSSO acknowledges the important role played by state educational agencies in adopting and evaluating new curriculum materials, and recommends that states communicate clearly about the quality of instructional materials to local educational agencies. It identifies several states that provide rubrics for local educational agencies to use when deciding on curriculum materials: Massachusetts, Mississippi, New Mexico, Rhode Island, and Tennessee are mentioned as having state educational agencies that offer comprehensive resources to local districts and schools regarding the selection of new materials. Additionally, we have observed progress in some state education agencies in offering toolkits to local education agencies during the curriculum evaluation and adoption cycle. One notable example is Massachusetts, where the Department of Elementary and Secondary Education has launched the expansive Curriculum Matters initiative, emphasizing high-quality instructional materials and aligned professional development.

CONSIDERATIONS FOR ENSURING ACCESS TO HIGH-QUALITY INSTRUCTIONAL MATERIALS:

1. State education agencies could consider offering a list of recommended high-quality instructional materials for each subject and encourage districts and schools to adopt materials solely from that list. Recommendations could be informed by various sources, including feedback from educators, independent reviews, and a comprehensive evaluation of materials that support differentiated instruction.

2. Consider providing local education agencies sufficient time to make decisions about material selection and supply them with additional resources that align with new standards before requiring commitment to the full adoption of new materials. The adoption of materials aligned with new standards could include a transition phase that permits districts to begin addressing the new standards without rushing into a poorly aligned set of new curriculum materials.

3. Consider providing teachers with professional development support that aligns with the adoption of new standards and instructional materials. Ongoing professional development on the use of instructional materials could offer consistent and cohesive messaging while creating opportunities for teachers to reflect and provide feedback.

CONCLUSION

A core value of our education system is that our nation's children, regardless of who they are, where they are from, or where they live, deserve access to a free public education that meets their needs. Access to qualified teachers who can deliver effective instruction for all students is key to fulfilling that aspiration. With contributions from mathematics experts, we have provided a landscape analysis of the three necessary access conditions with a focus on mathematics. We highlighted key challenges and several strategies currently employed to address them. We examined available information sequentially to answer the extent to which mathematics teachers (a) are available to fill open teaching positions, (b) have and use important knowledge, skills, and practices, and (c) have high-quality instructional materials available for use. While each condition, to be properly addressed, deserves its own report, we wanted to show the importance of addressing all three conditions. This report also served as the background document for the much shorter, public-facing brief on the CRPE website.

While the much-discussed results of the 2024 NAEP assessments showed the instructional sensitivity of students' mathematics achievement, they also indicated a growing gap between lower- and higher-performing students. Providing all students access to qualified mathematics teachers remains a critical policy lever for improving student performance. When the three access conditions are not met, student achievement may suffer; however, when they are met, there is potential for all students to excel mathematically. The three conditions discussed in this paper are fraught with challenges, some of which we have attempted to highlight. Naturally, there are other challenges we did not cover, and disentangling the various causes and effects in education is not straightforward.

Since the initial preparation of this report in the fall of 2024, significant events and changes have taken place in the U.S., with lasting impacts that directly affect the conditions mentioned above. Without reiterating the detailed takeaways for each access condition, the general answer to the access question is that we currently lack enough fully certified mathematics teachers for the various open positions, many of which are found in large suburban schools located in lower-income neighborhoods, serving predominantly historically underserved students. Strengthening the teacher pipeline remains the primary strategy for increasing the supply of future teachers, with alternative pathways into teaching representing a promising avenue for achieving this goal. Ensuring that the research community provides school leaders and professional organizations with guidance on the consensus for critical knowledge, skills, and practices to enhance the teaching and learning of mathematics remains essential to bridge the research-to-practice gap and put current debates into perspective. Even the decisions regarding the creation and selection of high-quality materials at both the state and local levels can benefit from independent guidance and support through evidence-based recommendations provided by the research community and shared by professional organizations.

The common denominator among all the challenges discussed and the strategies to address them is accurate data and rigorous research. Without the systematic collection of reliable data on our nation's schools, both within and across states, we are flying blind with nearly 50 million children onboard, unable to identify growing problems and emerging solutions before it is too late. Without leadership at the local, state, and national levels of our education system, we won't be able to exert the coordinated effort necessary to gather and examin the substantial amounts of data needed for understanding the state of the education system. Without independent, publicly funded education research, we will lose our ability to understand what is working in education and why and to develop solutions and innovations, and we risk abandoning the hard-earned progress we have made in educating one of the world's most diverse student populations. A lack of resources for and insights into the education system represents an unimaginably dull and cruel approach to improving public education, as the cost of any short-term savings falls on current and future generations of students and their teachers, along with the nation as a whole.

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